15/20

An-Najah National University; Engineering Faculty; Electrical Eng. Dept.
Electromagnetic Theory I (63251); Dr. Naser Abu-Zaid;

30-6-2009 **First Exam** **1.20 hr.**

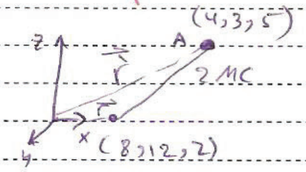
Student Name..Student No...............................*Section*.....10-11.....

*Insert your solutions **only** in the indicated places*

Question One) A $2\mu C$ point charge is located at A(4,3,5) in free space. Find E_ρ, E_φ, E_z at B(8,12,2). (زاوية الاتجاه) قس عند النقطة $\varphi = \tan^{-1}\frac{3}{4} = 36$

(6 marks)

① we find the ~~Act spherical~~ $\vec{E}$



$\vec{E} = \dfrac{Q}{4\pi\epsilon_0 R^3}\vec{R}$

$\vec{R} = \vec{r} - \vec{r}'$

$\vec{r} = 8\hat{a}_x + 12\hat{a}_y + 2\hat{a}_z$

$\vec{r}' = 4\hat{a}_x + 3\hat{a}_y + 5\hat{a}_z$

$\vec{R} = 4\hat{a}_x - 9\hat{a}_y + 3\hat{a}_z$

$R = \sqrt{16+81+9} = 10.29$

$\vec{E} = \dfrac{2\times10^{-6}}{(4)\times(3.14)(\epsilon_0)(\cancel{1089.54})} \quad 4\hat{a}_x - 9\hat{a}_y + 3\hat{a}_z$

1089.54

$= 66.02\hat{a}_x - 148.55\hat{a}_y + 49.51\hat{a}_z$

$E_\rho = \vec{E}\cdot\hat{a}_\rho = 66.02\cos(\varphi) - 148.55(+\sin\varphi) + 49.51(0) = 66.02\times(0.8) - 87.31 = \boxed{-34.449\,\hat{a}_\rho}$

$E_\varphi = \vec{E}\cdot\hat{a}_\varphi = 66.02(-\sin\varphi) - 148.55(\cos\varphi) + 0 = 38.8 - 120.17 = \boxed{-158.9\,\hat{a}_\varphi}$

$E_z = \boxed{49.51\,\hat{a}_z}$

5.5

Question Two) The value of **E** at $P(\rho = 2, \varphi = 40^o, z = 3)$ is given as $\mathbf{E} = 100\hat{\mathbf{a}}_\rho - 200\hat{\mathbf{a}}_\varphi + 300\hat{\mathbf{a}}_z \left(V/m\right)$. Determine the incremental work required to move $20\mu C$ a distance of $6\mu m$ in the direction of **E**. **(3 marks)**

1.5

$dW = -Q\,\vec{E}\,dL$

$Q = 20\mu$

$E = \checkmark$

$dL = 6\mu m$

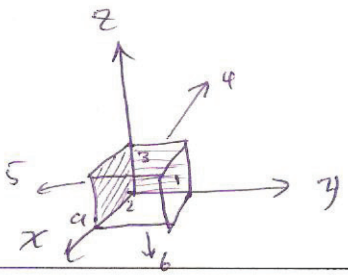
$= -20\mu(100\hat{a}_\rho - 200\hat{a}_\varphi + 300\hat{a}_z)\cdot 6\mu\,\hat{a}_L$

$= \boxed{-1.2\times10^{-8} + 2.4\times10^{-8} - 3.6\times10^{-8}}$ Joul

-2.4×10^{-8} Joul

(اتجاه الـ dL بعكس) $\hat{a}_E$ باتجاه وحدة $\hat{a}_E$ * تطلع

$Q\,dL\,\dfrac{|\vec{E}|^2}{|\vec{E}|} = Q\,dL\,|\vec{E}|$



النتيجة للنقطة x و y و z عند عاجزة التكامل

Question Three) Given the electric flux density $\mathbf{D} = 2xy\hat{\mathbf{a}}_x + x^2\hat{\mathbf{a}}_y + 6z^3\hat{\mathbf{a}}_z \left(C/m^2\right)$:

a) Use Gauss' law to evaluate the total charge enclosed in the volume $0 < x < a$, $0 < y < a$, $0 < z < a$.

b) Find the volume charge density at $x = \frac{a}{2}$, $y = \frac{a}{2}$, $z = \frac{a}{2}$. **(8 marks)**

$\nabla D = \rho_v$

a) gauss law

$\oint_s \vec{D} \cdot \vec{ds} = Q_{enc.}$

$\vec{D} = \checkmark$

$\vec{ds_1} = dx\,dz\;\hat{a}_y$

$ds_2 = dy\,dz\;\hat{a}_x$

$ds_3 = dx\,dy\;\hat{a}_z$

$ds_4 = dy\,dz\;-\hat{a}_x$

$ds_5 = dx\,dz\;-\hat{a}_y$

$ds_6 = dx\,dy\;\hat{a}_z$

$Q_1 = \oint \vec{D} \cdot \vec{ds_1} =$

$= \int_0^a\int_0^a x^2 \, dx\,dz$

$\frac{x^3}{3} z \Big|_0^a = \frac{a^3}{3} a = \frac{a^4}{3}$

$Q_2 = \oint \vec{D} \cdot ds_2$

$= \int_0^a\int_0^a 2xy \, dx\,dz$

$2y\left(\left(\frac{x^2}{2}\Big|_0^a\right) z\Big|_0^a\right) = \frac{a^4}{2}$

$Q_3 = \oint D \cdot ds_3$

$\int_0^a\int_0^a 6z^3 \, dx\,dy$

$= 6\; z^3\; xy$

$6a^3\,a^2 = 6a^5$

$Q_4 = \int_0^a\int_0^a -2xy \;\; dy\,dz$

$2x\frac{y^2}{2} z \Big|_{0\,0}^{a\,a}$

a^4

$Q_5 = \int_0^a\int_0^a -x^2 \; dx\,dz$

$= -\frac{x^3}{3} z \Big|_0^a = \frac{-a^4}{3}$

$Q_6 = \int_0^a\int_0^a -6z^3 \, dx\,dy$

$= -6z^3xy\Big|_0^a\Big|_0^a$

$= -6a^3 a^2 = -6a^5$

zero

$Q = Q_1 + \dots Q_6$

$= \boxed{0\;C}$

zero

5/6

b) using maxwel eqn.

في اتجاه الدايفرجنس

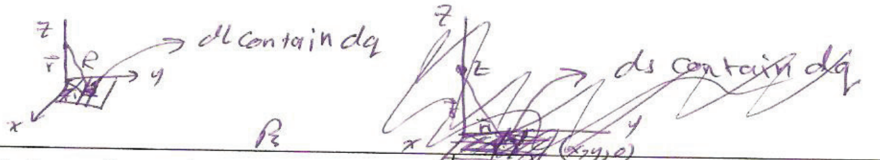
$|\nabla \cdot D| = \rho_v$

$|\nabla \cdot D| = 2y\,\hat{a}_x + 0\,\hat{a}_y + 18z\,\hat{a}_z$

$= 2y\,\hat{a}_x + 18z\,\hat{a}_z \Big|_p$

$= 2\frac{a}{2}\hat{a}_x + \frac{18a}{2}\hat{a}_z$

$\boxed{\sqrt{a^2 + 81a^2} \quad C/m^3}$



Question Four) A surface charge density $10^{-9}\left(C/m^2\right)$ (assumed uniform) exists for $z=0$, $-50 \le x \le 50$, $-0.05 \le y \le 0.05$. Using reasonable approximations, find $\mathbf{E}(0,0,z)$ when $z=10^4$. **(3 marks)**

y is small we can assume ~~it like line~~ like po...

$$d\vec{E} = \frac{dq}{4\pi\epsilon_0} \cdot \frac{\vec{R}}{R^3}$$

$dq = \rho_s \cdot ds \rightarrow dx\,dy$

$\vec{R} = \vec{r} - \vec{r}'$

$\vec{r} = z\hat{a}_z$

$\vec{r}' = x\hat{a}_x + y\hat{a}_y$

$\vec{R} = z\hat{a}_z - x\hat{a}_x - y\hat{a}_y$

$R = \sqrt{x^2+y^2+z^2}$

$$d\vec{E} = \frac{\rho_s\, dx\,dy}{4\pi\epsilon_0} \cdot \frac{z\hat{a}_z - x\hat{a}_x - y\hat{a}_y}{\left(\sqrt{x^2+y^2+z^2}\right)^3}$$

$$\vec{E} = \int_{-0.05}^{0.05}\int_{-50}^{50} \frac{\rho_s\, dx\,dy}{4\pi\epsilon_0} \cdot \frac{z\hat{a}_z - x\hat{a}_x - y\hat{a}_y}{\left(\sqrt{x^2+y^2+z^2}\right)^3}$$ → not function of ...

$$\vec{E} = \frac{\rho_s}{4\pi\epsilon_0}\left(\int_s \frac{z\hat{a}_z\, dx\,dy}{\left(\sqrt{x^2+y^2+z^2}\right)^3} - \int_s \frac{x\hat{a}_x\, dx\,dy}{\left(\sqrt{x^2+y^2+z^2}\right)^3} - \int_s \frac{y\hat{a}_y\, dx\,dy}{\left(\sqrt{x^2+y^2+z^2}\right)^3}\right)$$

$$= \frac{\rho_s}{4\pi\epsilon_0}\left(-\hat{a}_x - \hat{a}_y + \hat{a}_z\right)\Big($$

take differential length (dL) Char...

$dq = \rho_L \cdot dL \rightarrow dx$

$\vec{R} = \vec{r} - \vec{r}'$

$r = z\hat{a}_z$

$\vec{r}' = x\hat{a}_x$

$R = z\hat{a}_z - x\hat{a}_x$

$R = \sqrt{z^2+x^2}$

$$d\vec{E} = \frac{\rho_s\, dx}{4\pi\epsilon_0} \cdot \frac{z\hat{a}_z - x\hat{a}_x}{\left(\sqrt{z^2+x^2}\right)^3}$$

$$\int_L \frac{\rho_s\, dx}{4\pi\epsilon_0} \cdot \frac{z\hat{a}_z - x\hat{a}_x}{\left(\sqrt{z^2+x^2}\right)^3}$$

$$\frac{\rho_s\, a_z}{4\pi\epsilon_0} \cdot \frac{z}{\sqrt{z^2+x^2}}$$

$$\frac{\rho_s\, a_z}{4\pi\epsilon_0}\left(\int_{-50}^{50} \frac{z}{\sqrt{z^2+x^2}}\, dx + \int_{-50}^{50} \frac{x}{\sqrt{z^2+x^2}}\, dx\right)$$

$$d\vec{E} = \frac{\rho_s \cdot ds}{4\pi\epsilon_0\,(10^4)^3}\,\hat{a}_z$$

$$= \frac{10^{-9}(100 \times 0.1)}{4\pi\epsilon_0\,(10^4)^3}\,\hat{a}_z$$

$\boxed{\cancel{89.98}\ \hat{a}_z}$

$\boxed{8.99 \times 10^{-13}\,\hat{a}_z}$

1.5

لیش ds

E لازم يكون على

Good Luck

Electromagnetic Theory I (Equation) Sheet and constants

Divergence

Rectangular
$$\nabla \bullet \mathbf{D} = \frac{\partial}{\partial x}(D_x) + \frac{\partial}{\partial y}(D_y) + \left(\frac{\partial D_z}{\partial z}\right)$$

Cylindrical
$$\nabla \bullet \mathbf{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial}{\partial \varphi}(D_\varphi) + \left(\frac{\partial D_z}{\partial z}\right)$$

Spherical
$$\nabla \bullet \mathbf{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \theta}(D_\theta \sin\theta) + \frac{1}{r\sin\theta}\left(\frac{\partial D_\varphi}{\partial \varphi}\right)$$

Gradient

Rectangular
$$\nabla V = \frac{\partial V}{\partial x}\hat{\mathbf{a}}_x + \frac{\partial V}{\partial y}\hat{\mathbf{a}}_y + \frac{\partial V}{\partial z}\hat{\mathbf{a}}_z$$

Cylindrical
$$\nabla V = \frac{\partial V}{\partial \rho}\hat{\mathbf{a}}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \varphi}\hat{\mathbf{a}}_\varphi + \frac{\partial V}{\partial z}\hat{\mathbf{a}}_z$$

Spherical
$$\nabla V = \frac{\partial V}{\partial r}\hat{\mathbf{a}}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\mathbf{a}}_\theta + \frac{1}{r\sin\theta}\left(\frac{\partial V}{\partial \varphi}\right)\hat{\mathbf{a}}_\varphi$$

Curl

Rectangular
$$\nabla \times \mathbf{H} = \left[\frac{\partial(H_z)}{\partial y} - \frac{\partial(H_y)}{\partial z}\right]\hat{\mathbf{a}}_x + \left[\frac{\partial(H_x)}{\partial z} - \frac{\partial(H_z)}{\partial x}\right]\hat{\mathbf{a}}_y + \left[\frac{\partial(H_y)}{\partial x} - \frac{\partial(H_x)}{\partial y}\right]\hat{\mathbf{a}}_z$$

Cylindrical
$$\nabla \times \mathbf{H} = \left[\frac{1}{\rho}\frac{\partial(H_z)}{\partial \varphi} - \frac{\partial(H_\varphi)}{\partial z}\right]\hat{\mathbf{a}}_\rho + \left[\frac{\partial(H_\rho)}{\partial z} - \frac{\partial(H_z)}{\partial \rho}\right]\hat{\mathbf{a}}_\varphi + \frac{1}{\rho}\left[\frac{\partial(\rho H_\varphi)}{\partial \rho} - \frac{\partial(H_\rho)}{\partial \varphi}\right]\hat{\mathbf{a}}_z$$

Spherical
$$\nabla \times \mathbf{H} = \frac{1}{r\sin\theta}\left[\frac{\partial(H_\varphi \sin\theta)}{\partial \theta} - \frac{\partial(H_\theta)}{\partial \varphi}\right]\hat{\mathbf{a}}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial(H_r)}{\partial \varphi} - \frac{\partial(rH_\varphi)}{\partial r}\right]\hat{\mathbf{a}}_\theta + \frac{1}{r}\left[\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial(H_r)}{\partial \theta}\right]\hat{\mathbf{a}}_\varphi$$

Integrals

$$\int \sin(x)dx = -\cos(x)$$

$$\int \frac{dx}{\sqrt{1+x^2}} = \tan^{-1}(x)$$

$$\int \frac{dx}{ax^2+c} = \frac{1}{\sqrt{ac}}\tan^{-1}\left(x\sqrt{\frac{a}{c}}\right)$$

$$\int \cos(x)dx = \sin(x)$$

$$\int \frac{dx}{|x|\sqrt{x^2-1}} = \sec^{-1}(x)$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x)$$

$$\int \frac{dx}{p^2-x^2} = \frac{1}{2p}\ln\left|\frac{p+x}{p-x}\right|$$

Constants

$$\mu_o = 4\pi \times 10^{-7}\left(\frac{H}{m}\right)$$

$$\varepsilon_o = \frac{1}{\mu_o^2 c^2} = \frac{10^{-9}}{36\pi} = 8.854 \times 10^{-12}\left(\frac{F}{m}\right)$$

$$c = 3 \times 10^8\left(\frac{m}{s}\right)$$