Angle Modulation:

Amplitude modulation is used to vary , slowly, the amplitude of a sinusoidal carrier wave in accordance with the base band signal.

Angle modulation is used to vary the angle of the carrier wave according to the base band signal .(amplitude of c(t) is maintained constant)

Angle modulation provide better discrimination against noise & interference than Amplitude modulation but this requires more transmission Band width .

(exchanging channel B.W for improved noise performance Not possible with AM).

Definitions :

$y(t) = \cos(6\pi t) \quad \rightarrow \quad f = 3Hz.$

$y(t) = \cos(6\pi t e^{-t}) \quad \rightarrow \quad f = ??.$

Angle modulated signal is given as :

$s(t) = A_c \cos\big(\theta_i(t)\big)$ A_c: carrier amplitude

$\theta_i(t)$: a function of $m(t)$.

When $\theta_i(t)$ changes by 2π we have complete oscillation .

The average frequency in Hz over an interval $t \rightarrow t + \Delta t$ is

$$f_{\Delta t}(t) = \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi\Delta t}$$ average frequency.

Define the instantaneous frequency of $s(t)$ as :

$$f_i(t) = \lim_{\Delta t \rightarrow 0} f_{\Delta t}(t)$$

$$[\ f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}\ ]$$ instantaneous frequency.

For un modulated carrier :

$$\theta_i(t) = 2\pi f_c t \ + \ \varphi_c$$

Angle modulation means varying $\theta_i(t)$ in some manner with the message baseband signal $m(t)$. (infinite number of ways)

1) Phase Modulation (PM) :

The $\theta_i(t)$ is varied linearly with m(t) as :

$\theta_i(t) = 2\pi f_c t \ + \ k_p m(t)$

Where :

$2\pi f_c t \ \equiv angle\ of\ un\ modulated\ carrier$

$k_p \equiv phase\ sensitivity\ of\ the\ modulator\ (rad/volt)$

$$\rightarrow \boxed{s(t) = A_c \ \cos\,[2\pi f_c t \ + \ k_p m(t)]}$$ PM

2) Frequency Modulation (FM) :

The instantaneous frequency $f_i(t)$ is varied linearly with $m(t)$.

$$f_i(t) = f_c + \ k_f m(t) \qquad \rightarrow (*)$$

Where :

$f_c \equiv \ frequency\ of\ the\ un\ modulated\ carrier$

$k_f \equiv frequency\ sensitivity\ of\ the\ modulator\ (Hz/volt)$

$$\int * \rightarrow \quad 2\pi \int \ f_i(t) = 2\pi\ [\int(\ f_c + \ k_f m(t))\ \ dt\ ]$$

$$\therefore \theta_i(t) = 2\pi f_c t \ + 2\pi\ k_f \int_0^t \ m(T)dT$$

$$\rightarrow \boxed{s(t) = A_c \ \cos\,[2\pi f_c t \ + 2\pi\ k_f \int_0^t \ m(T)dT \ ]}$$ FM

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$$

$$\therefore \int f_i(t) = \frac{1}{2\pi} \int \frac{d\theta_i(\tau)}{d\tau} \, dt$$

$$\therefore \left[\, 2\pi \int f_i(t) = \theta_i(t) \,\right].$$

$\rightarrow \quad s(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right]$ PM

$\rightarrow \quad s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(T)dT\right]$ FM

1) The zero crossing of PM or FM signals have No regularity in their spacing (see Bessel function).

2) The envelope of a PM or FM signal is constant (equal the carrier amplitude but the envelope of AM depends on the message signal).

FM & PM can be generated from each other .

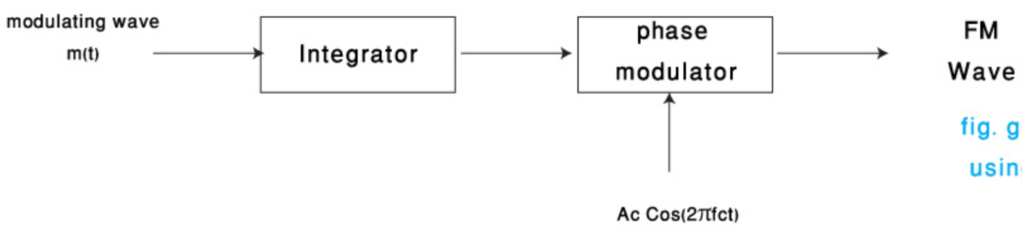


fig. generating an FM using PM modulator

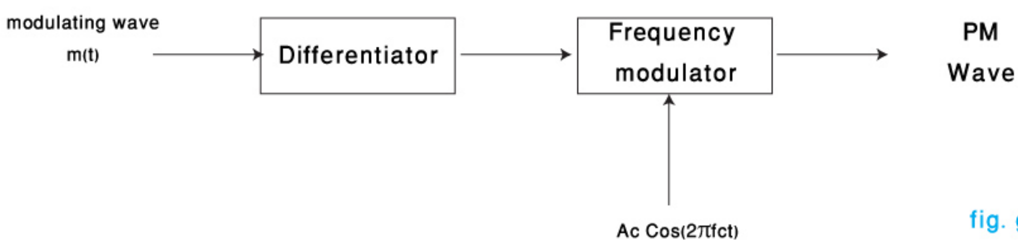
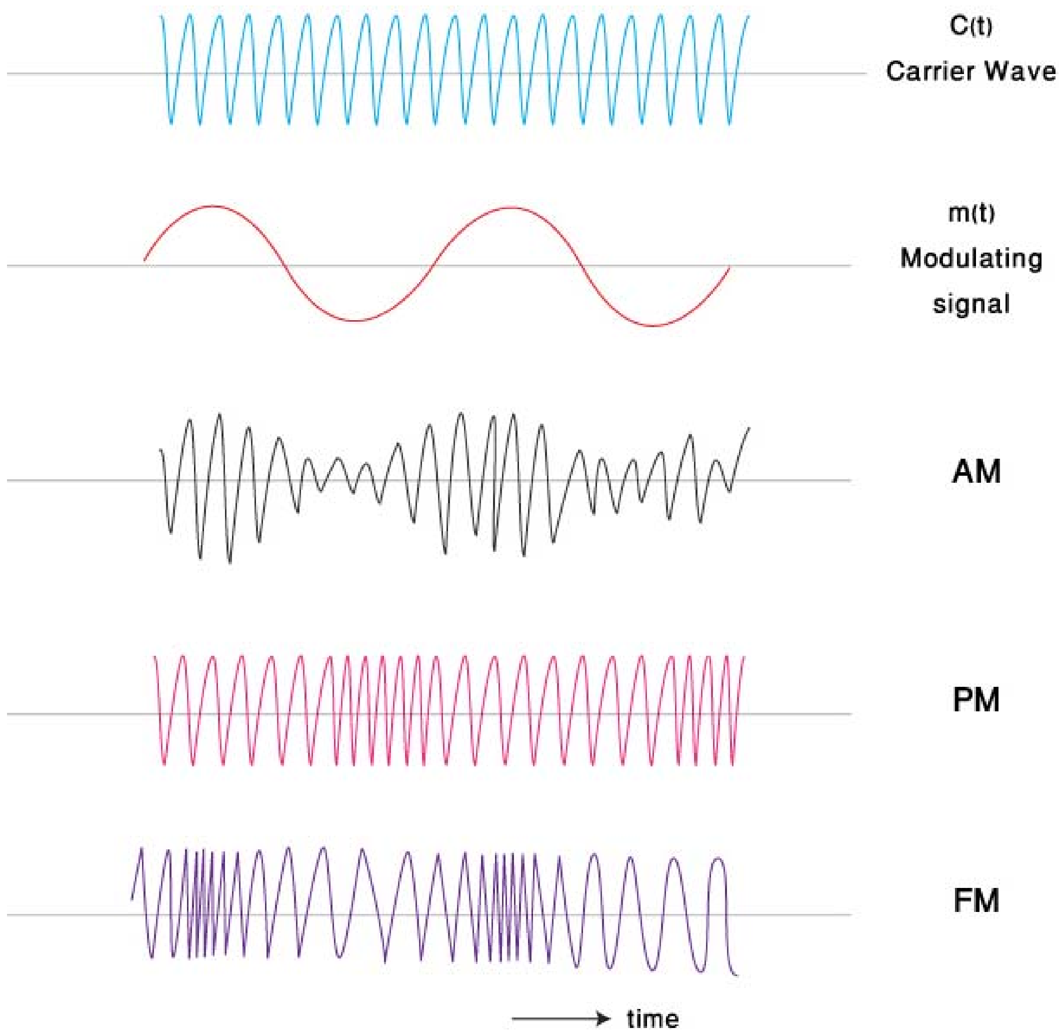


fig. generating a PM using FM modulator

PM & FM all properties of one can be deduced from the other.



Frequency Modulation:

$s(t) = A_c \ \cos\,[2\pi f_c t \ + 2\pi \ k_f \int_0^t \ m(T)dT \ ]$ FM

- Non linear modulation process
- More difficult than AM

$*\, Spectral\ analysis\ of\ FM\ signal\,.$

Consider a modulating signal

$m(t) = A_m \cos(2\pi f_m t)$

Then the instantaneous frequency

$f_i(t) = f_c + k_f A_m \cos(2\pi f_m t)$

$f_i(t) = f_c + \Delta f \cos(2\pi f_m t)$ $\qquad\qquad$ f_m: $modulating\ frequency.$

$\boldsymbol{\Delta f = K_f A_m}$: **Frequency deviation** representing the maximum departure of the instantaneous frequency of the FM signal from the carrier frequency f_c **.**

$$\Delta f \begin{cases} \textbf{depends on Am} \\ \textbf{independent from Fm} \end{cases}$$

$\theta_i(t) = 2\pi \int_0^t f_i(T)dT$

$\theta_i(t) = 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t)$

$\frac{\Delta f}{f_m}$: $modulation\ index\ of\ the\ FM\ signal.$

Let $\beta = \frac{\Delta f}{f_m}$ (rad)

$\therefore$ [$\theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t)$]

$\boldsymbol{\beta}$**:** represents the **phase deviation** of the FM signal (Max. departure of $\theta_i(t)$ from $2\pi f_c t$)

FM :

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$

Cases of FM :

1) Narrow band FM , $\beta < 1 \quad rad.$

2) Wide band FM , $\beta > 1 \quad rad.$