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Electromagnetic Theory I (63251); Dr. Naser Abu-Zaid;

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Student Name.................................Student No.............................Section........

*Insert your solutions in the indicated places **only***

Question One) A uniform surface charge density of $20\ ^{nC}/_{m^2}$ is present on the spherical surface $r = 0.6cm$ in free space.

a) Find the absolute potential at $P(1cm, 25^o, 50^o)$. **(2 marks)**

b) Find V_{AB} given points $A(2cm, 30^o, 60^o)$ and $B(2cm, 45^o, 90^o)$. **(2 marks)**

(a) $V = \iint_S \frac{\rho_s}{4\pi\epsilon_0 r^2} \cdot ds$ r cos ∮ sinθ 0.015

$4\pi r^2$ $\rho_s S$

$ds = r^2 \sin\theta \, d\phi \, d\theta$

$\Rightarrow V = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{\rho_s}{4\pi\epsilon_0 r^2} r^2 \sin\theta \, d\phi \, d\theta$

$= \int_0^{\pi}\int_0^{2\pi} \frac{\rho_s \sin\theta}{4\pi\epsilon_0} d\phi \, d\theta$

$= \frac{\rho_s}{4\pi\epsilon_0}\int_0^{\pi}\int_0^{2\pi} \sin\theta \, d\phi \, d\theta$

3

$\int_0^{2\pi} -\cos\theta\Big]_0^{\pi} d\phi$

$(1)(\phi)\Big]_0^{2\pi} = \boxed{2\pi} V$

~~$V_{AB} = \int \frac{\rho_s}{4\pi\epsilon_0} \ldots A \ldots B$~~

~~$= \iint \frac{\rho_s \, ds}{4\pi\epsilon_0}$~~

(b) same r from the P

$\Rightarrow \rho_s$ uniform $\Rightarrow \boxed{V_{AB} = 0}$

Question Two) A Dipole having a moment $\mathbf{p} = 3\hat{\mathbf{a}}_x - 5\hat{\mathbf{a}}_y + 10\hat{\mathbf{a}}_z (nC.m)$ is located at $A(1, 2, -4)$ in free space. Find V at $B(2, 3, 4)$ **(3 marks)**

$V = \frac{Q \cdot d \cos\theta}{4\pi\epsilon_0 r^2} \Rightarrow \frac{|P|}{4\pi\epsilon_0 r^2} = \frac{\sqrt{9+25+100}}{4\pi\epsilon_0 r^2} = \frac{11.57 \times 10^{-9}}{1.11 \times 10^{-10} r^2}$

$r = (2-1)\hat{a}_x + (3-2)\hat{a}_y + (4+4)\hat{a}_z$

$= \hat{a}_x + \hat{a}_y + 8\hat{a}_z$

$= \sqrt{1+1+64} = 8.12$

2

$\Rightarrow V = \frac{11.57 \times 10^{-9}}{1.11 \times 10^{-10} \times 8.12} = 1.28 \times 10^{-16} \times 10^{-9} = 12.8 V$

solution $V = \frac{\vec{P} \cdot (\vec{r} - \vec{r}')}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|^3}$

Question Three) A coaxial conductor has radii $a = 0.8mm$ and $b = 3mm$ and a dielectric material in between for which $\varepsilon_r = 2.56$. If $\mathbf{P} = \frac{2}{\rho}\hat{\mathbf{a}}_\rho \left(nC/_{m^2}\right)$ in the dielectric:

a) Find **D** and **E** as functions of ρ. **(2 marks)**

b) Find V_{ab}. **(2 marks)**

$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$, $\boxed{\vec{E} = \frac{\vec{D}}{\chi_0 \epsilon_0}}$

$\vec{P} = \chi_e \epsilon_0 \vec{E}$

solutions formulas

$\vec{E} = \frac{\vec{P}}{\chi_0 \epsilon_0}$

~~$|D| = \rho_s$ Boundary~~

~~$\vec{D}_{coaxial} = \frac{a\rho_s}{\rho} \hat{a}_\rho = \frac{(0.8m)(|D|)}{\rho}$~~

$\frac{2}{\rho} \hat{a}_\rho \times 10^{-9} = (2.56 - 1)(\epsilon_0) \vec{E}$

$\boxed{\vec{E} = \frac{1.44 \times 10^{11}}{\rho} \hat{a}_\rho} \Rightarrow \boxed{D = \frac{1.44 \times 10^{11}}{\rho} \epsilon_0 \hat{a}_\rho}$

$V_{ab} = \int_a^b \vec{E} \cdot d\vec{l}$ just normal

$= \int_a^b \frac{1.44 \times 10^{11}}{\rho} \hat{a}_\rho \cdot d\rho\, \hat{a}_\rho$

$1.44 \times 10^{11} * 2.2m$

$\boxed{316.8 \times 10^{6}\ V}$

3.5

Question Four) Given The current density $J = -10^4\left[\sin(2x)e^{-2y}\hat{\mathbf{a}}_x + \cos(2x)e^{-2y}\hat{\mathbf{a}}_y\right]\left(kA/_{m^2}\right)$

a) Find the total current crossing the plane $z = 3$ in the $\hat{\mathbf{a}}_z$ direction in the region $0 \le x \le 1$, $1.5 \le y \le 2.5$. **(1.5 marks)** xy plane

b) Find the total current leaving the region $r = 5cm$. **(1.5 marks)**

$I = \iint_s \vec{J} \cdot d\vec{s}$

$ds = dx\,dy\,\hat{a}_z$ → when we multiply with a_x & a_y → 0

→ there is no $\hat{a}_z$ component to J

$\int_{1.5}^{2.5}\int_0^1 -10^4 [\sin(2x) e^{-2y} \hat{a}_x + \cos(2x) e^{-2y} \hat{a}_y] \cdot dx\,dy\,\hat{a}_z = 0\,\hat{a}_z$

2

trick

$I = \iiint_s \vec{J} \cdot d\vec{s} = \vec{J} \iint_s d\vec{s}$

$= \vec{J} \cdot \vec{S}$

$= \vec{J} \cdot 4\pi r^2 \hat{a}_r$

solution

$I = \oint_s J \cdot ds = \iiint_v (\nabla \cdot J)\, dv$

$\nabla \cdot J =$

$\boxed{I = 0}$

$= 10^4 [\sin(2x) e^{-2y} \hat{a}_x + \cos(2x) e^{-2y} \hat{a}_y] \cdot 4\pi r^2 \hat{a}_r$

$[(10^4)(4)\pi(25)(\sin 2x) e^{-2y} \hat{a}_x \cdot \hat{a}_r]$ → cosθ sinϕ 2

$+ \{(10^4)(4)\pi(25)(\cos 2x) e^{-2y} \hat{a}_y \cdot \hat{a}_r]$ → cosθ cosϕ

Question Five) Uniform surface charge densities of $6\left(nC/m^2\right)$ and $2\left(nC/m^2\right)$ are prese[nt at]
$r = 2cm$ and $r = 6cm$ respectively, in free space. Assume $V = 0$ at $r = 4cm$, and calc[ulate]
V at $r = 7cm$. **(6 marks)**

$\rho_{s_1} = 6\,nC/m^2 \longrightarrow r = 2cm$

$\rho_{s_2} = 2\,nC/m^2 \longrightarrow r = 6\,cm$

$$\boxed{V = \frac{Q}{4\pi\epsilon_0 r^2}}$$

$$0 = \int_s \frac{\rho_{s_1} \cdot ds_1}{4\pi\epsilon_0 r_1^2} + \int_s \frac{\rho_{s_2} \cdot ds_2}{4\pi\epsilon_0 r_2^2} \Rightarrow r = 4$$

6nC, 2nC

$(\vec{r}_1 - \vec{r}_2) \to (0.02)^2$ $\quad (0.02)^2 \quad (\vec{r}_1 - \vec{r}_2)$

3

$$\frac{6nc}{4\pi\epsilon_0 (0.02)^2}\int_s ds_1 + \frac{2nc}{4\pi\epsilon_0 (0.02)^2}\int_s ds_2 = 0$$

$$S_1 \frac{6nc}{\cancel{4\pi\epsilon_0 (0.02)^2}} = \frac{-2nc}{\cancel{4\pi\epsilon_0 (0.02)^2}} S_2$$

$$6S_1 = -2S_2$$

$$\boxed{3S_1 = -S_2}$$

$$V_{at\,r=7} \Rightarrow \int_s \frac{\rho_{s1} \cdot ds_1}{4\pi\epsilon_0 (0.05)^2} + \int_s \frac{\rho_{s2} \cdot ds_2}{4\pi\epsilon_0 (0.01)^2}$$

$$-\frac{S_2}{3} \frac{6nc}{4\pi\epsilon_0 (0.05)^2} + {+S_2}\frac{2nc}{4\pi\epsilon_0 (0.01)^2}$$

$$V = S_2 \left(\frac{1nc}{2\pi\epsilon_0\, 2.5 \times 10^{-3}} + \frac{nc}{2\pi\epsilon_0 \times 10^{-4}} \right)$$

Good Luck

Electromagnetic Theory I (Equation) Sheet and constants

Divergence

Rectangular

$$\nabla \bullet \mathbf{D} = \frac{\partial}{\partial x}(D_x) + \frac{\partial}{\partial y}(D_y) + \left(\frac{\partial D_z}{\partial z}\right)$$

Cylindrical

$$\nabla \bullet \mathbf{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial}{\partial \varphi}(D_\varphi) + \left(\frac{\partial D_z}{\partial z}\right)$$

Spherical

$$\nabla \bullet \mathbf{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \theta}(D_\theta \sin\theta) + \frac{1}{r\sin\theta}\left(\frac{\partial D_\varphi}{\partial \varphi}\right)$$

Gradient

Rectangular

$$\nabla V = \frac{\partial V}{\partial x}\hat{\mathbf{a}}_x + \frac{\partial V}{\partial y}\hat{\mathbf{a}}_y + \frac{\partial V}{\partial z}\hat{\mathbf{a}}_z$$

Cylindrical

$$\nabla V = \frac{\partial V}{\partial \rho}\hat{\mathbf{a}}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \varphi}\hat{\mathbf{a}}_\varphi + \frac{\partial V}{\partial z}\hat{\mathbf{a}}_z$$

Spherical

$$\nabla V = \frac{\partial V}{\partial r}\hat{\mathbf{a}}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\mathbf{a}}_\theta + \frac{1}{r\sin\theta}\left(\frac{\partial V}{\partial \varphi}\right)\hat{\mathbf{a}}_\varphi$$

Curl

Rectangular

$$\nabla \times \mathbf{H} = \left[\frac{\partial(H_z)}{\partial y} - \frac{\partial(H_y)}{\partial z}\right]\hat{\mathbf{a}}_x + \left[\frac{\partial(H_x)}{\partial z} - \frac{\partial(H_z)}{\partial x}\right]\hat{\mathbf{a}}_y + \left[\frac{\partial(H_y)}{\partial x} - \frac{\partial(H_x)}{\partial y}\right]\hat{\mathbf{a}}_z$$

Cylindrical

$$\nabla \times \mathbf{H} = \left[\frac{1}{\rho}\frac{\partial(H_z)}{\partial \varphi} - \frac{\partial(H_\varphi)}{\partial z}\right]\hat{\mathbf{a}}_\rho + \left[\frac{\partial(H_\rho)}{\partial z} - \frac{\partial(H_z)}{\partial \rho}\right]\hat{\mathbf{a}}_\varphi + \frac{1}{\rho}\left[\frac{\partial(\rho H_\varphi)}{\partial \rho} - \frac{\partial(H_\rho)}{\partial \varphi}\right]\hat{\mathbf{a}}_z$$

Spherical

$$\nabla \times \mathbf{H} = \frac{1}{r\sin\theta}\left[\frac{\partial(H_\varphi \sin\theta)}{\partial \theta} - \frac{\partial(H_\theta)}{\partial \varphi}\right]\hat{\mathbf{a}}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial(H_r)}{\partial \varphi} - \frac{\partial(rH_\varphi)}{\partial r}\right]\hat{\mathbf{a}}_\theta + \frac{1}{r}\left[\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial(H_r)}{\partial \theta}\right]\hat{\mathbf{a}}_\varphi$$

Integrals

$$\int \sin(x)dx = -\cos(x) \qquad \int \frac{dx}{\sqrt{1+x^2}} = \tan^{-1}(x) \qquad \int \frac{dx}{ax^2+c} = \frac{1}{\sqrt{ac}}\tan^{-1}\left(x\sqrt{\frac{a}{c}}\right)$$

$$\int \cos(x)dx = \sin(x) \qquad \int \frac{dx}{|x|\sqrt{x^2-1}} = \sec^{-1}(x)$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x) \qquad \int \frac{dx}{p^2-x^2} = \frac{1}{2p}\ln\left|\frac{p+x}{p-x}\right|$$

Constants

$$\mu_o = 4\pi \times 10^{-7}\left(\frac{H}{m}\right) \qquad \varepsilon_o = \frac{1}{\mu_o^2 c^2} = \frac{10^{-9}}{36\pi} = 8.854 \times 10^{-12}\left(\frac{F}{m}\right) \qquad c = 3 \times 10^8\left(\frac{m}{s}\right)$$