$\frac{9.5}{19}$

An-Najah National University
Engineering Faculty
Electrical Eng. Dept.
Electromagnetic Theory I (63251)
Dr. Naser Abu-Zaid
Second Exam

22-7-2008 1 hr.

Student Name……… *Student No*………………… *Section*…10 – 11

*Insert your answers **only** in the indicated places*

Question one)

A) An electron $(Q = 1.602 \times 10^{-19}\ C)$ leaves the cathode of a cathode ray tube (CRT) and travels in a uniform electrostatic field toward the anode, which is at a potential $V = 500\,(V)$ with respect to the cathode. What is the work done by the electrostatic field in moving the electron from the cathode to the anode? **(2marks)**

2

$V \triangleq \frac{W}{Q} \Rightarrow 500 \triangleq \frac{W}{1.602 \times 10^{-19}}$ $\boxed{W = 8.01 \times 10^{-17}\ \text{Joule}}$

B) In part **(A)**, what is the electric field strength $(|\mathbf{E}|)$ if the distance between the anode and cathode is 10cm. **(2marks)**

0.5

$w = \frac{\epsilon_0}{2} \int |E|\, dl$ $8.01 \times 10^{-17} = \frac{10^{-9}}{72\pi} |E| \times 10 \times 10^{-2}$ $|\vec{E}| = 1.8 \times 09 \times 10^{-4}$

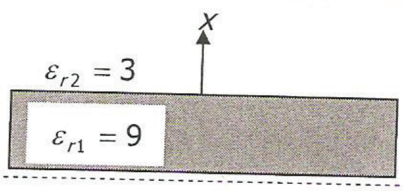
C) The electrostatic potential $V = \frac{2 \times 10^{-3}}{\sqrt{\varepsilon_o}} x$, exists in a region in the shape of a parallelogram of size $10 \times 10 \times 1cm$. What is the electrostatic energy stored in this region. **(3marks)**

0.5

$V = \frac{W}{Q}$ $W_E = \frac{2 \times 10^{-3}\, x}{\sqrt{\epsilon_0}} * Q$ $W_E = \frac{1}{2} \int \rho_v V_{(x,y,z)}\, dv$

D) Two dielectric regions have planar interface at x=0. The field in region 1 is $\mathbf{E}_1 = 4\hat{\mathbf{a}}_x - 10\hat{\mathbf{a}}_y$. What are the electric field intensity $\mathbf{E}_2$ and polarization vector $\mathbf{P}_2$. **(3marks)**

1.5



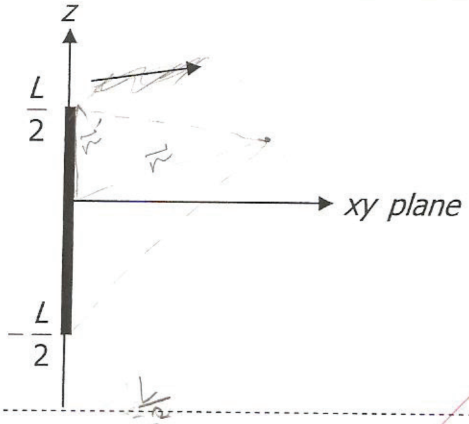
$\hat{a}_n \cdot (\vec{D}_2 - \vec{D}_1) = 0$
$\vec{D}_{n2} - D_{n1} = 0$
$D_{n2} = D_{n1} \Rightarrow$

$\Rightarrow \epsilon_2 \vec{E}_{n2} = \epsilon_1 \vec{E}_{n1}$
$3 (4 \hat{a}_x) = 9 (\ \ )$
$\vec{E}_{n1} = \frac{4}{3} \hat{a}_x$!!??

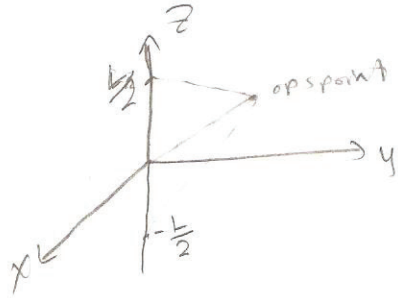
$\hat{a}_n \times (\vec{E}_2 - \vec{E}_1) = 0$
$\vec{E}_{t2} - \vec{E}_{t1} = 0$
$\vec{E}_{t2} = \vec{E}_{t1}$
$-10 \hat{a}_y = E_{t1}$

$\vec{E}_1 = \frac{4}{3}\hat{a}_x - 10 \hat{a}_y$ 12
$\vec{P}_2 = \chi_{e_2} \epsilon_0 \vec{E}_2 = (\epsilon_{r2} - 1)\epsilon_0 \vec{E}_2$
$= 2 \epsilon_0 E_2$

Question Two) A short segment of length L is charged with a line charge density $\rho_l \left(C/m\right)$ as shown in the figure. Set up the integral needed to calculate the potential at a general point in space. **(4marks)**



2.5



$$V_2 = \int_0^{L/2} \frac{\rho_l\, dl}{4\pi\epsilon_0 |\vec r - \vec r'|}$$

$$V_1 = \int_{-L/2}^{0} \frac{\rho_l\, dl}{4\pi\epsilon_0 |\vec r - \vec r'|}$$

$$\vec r = r\,\hat a_r$$

$$\vec r' = \frac{L}{2}\,\hat a_z$$

$$|\vec r - \vec r'| = |r\,\hat a_r - \tfrac{L}{2}\,\hat a_z|$$

$$V_2 = \int_0^{L/2} \frac{\rho_l\, dl}{4\pi\epsilon_0 |r\,\hat a_r - \frac{L}{2}\hat a_z|}$$

$$\vec r = \vec r\,\hat a_r$$

$$\vec r' = -\frac{L}{2}\,\hat a_z$$

$$|\vec r - \vec r'| = |r\,\hat a_r + \tfrac{L}{2}\,\hat a_z|$$

$$V_1 = \int_0^{L/2} \frac{\rho_l\, dl}{4\pi\epsilon_0 |r\,\hat a_r + \frac{L}{2}\hat a_z|}$$

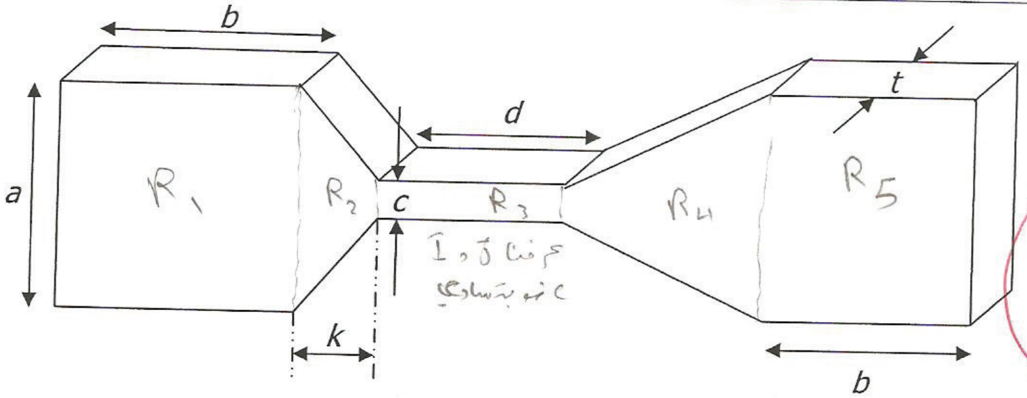
$$V = \int_{-L/2}^{0} \frac{\rho_l\, dl}{4\pi\epsilon_0 |r\,\hat a_r - \frac{L}{2}\hat a_z|} + \int_0^{L/2} \frac{\rho_l\, dl}{4\pi\epsilon_0 |r\,\hat a_r + \frac{L}{2}\hat a_z|}$$

Question Three) Fuses rely on melting of a piece of conductor when current in the fuse exceeds a given value. A fuse is made of copper in the shape shown in the figure. The Two large sections are intended to clamp the fuse while the narrow section is the actual fuse. Copper can carry a safe current density (J) of $10^8\left(A/m^2\right)$. Above this current, copper will melt.

a) Design the width of the fusing section so that it will break at currents above $I = 20A$.

b) Calculate the total resistance of the fuse for the dimensions given in the figure and with the width found in part **(a)**.

(5marks)



2.5

$a = 4mm,\ b = 50mm,\ d = 20mm,\ k = 20mm,\ t = 0.1mm,\ \sigma = 5.7\times10^7\ S/m$

a) $I = \int_s \bar{J}\, ds$

$\int ds = \cancel{d}^{\,t} \times c$

$\int ds = \cancel{20}^{\,0.1}\times10^{-3}\times C$

$I = 10^8 \int ds$

$20 = 10^8 \int ds$

$\cancel{20}\times10^{-8} = \cancel{20}\times10^{-3}\times C$

$\boxed{C = 1\times10^{-5}\ m}$

2 mm

b) $R = \dfrac{V_{ab}}{I} = \dfrac{-\int \bar{E}\, dl}{\int_s \sigma . ds}$ E is uniform and is constant

$R_1 = \dfrac{L}{\sigma A} = \dfrac{50\ mm}{5.7\times10^7\times(0.1\times10^{-3}\times 4\times10^{-3})}$

$= 2.193\ m\Omega$

and $R_5 = 2.193\ m\Omega$

$R_3 = \dfrac{L}{\sigma A} = \dfrac{20\ mm}{5.7\times10^7\times(0.1\times10^{-3}\times\cancel{4\times10^{-5}})}$ 2 mm

$R_3 = 0.3508\ \Omega$

$R_2 = \dfrac{L}{\sigma A} = \dfrac{20\ mm}{5.7\times10^7\times(7.9995\times10^{-6})} = 43.86\ \mu\Omega$

and $R_4 = 43.86\ \mu\Omega$

$A = \int_c^a t\times L\, dL$

$= 0.1\times10^{-3}\int_{1\times10^{-5}}^{4\times10^{-3}} L\, dL =$

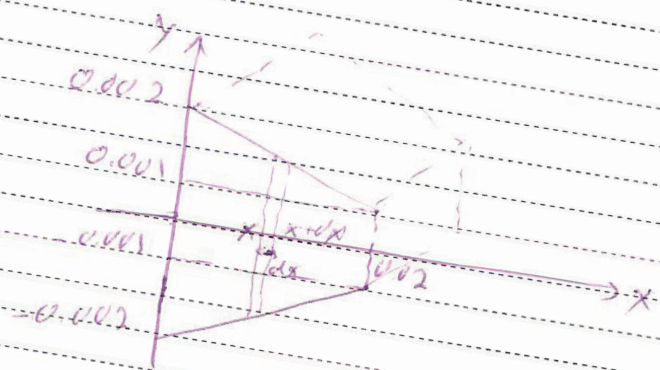
$= 0.1\times10^{-3}\ \frac{1}{2}L^2\Big|_{1\times10^{-5}}^{4\times10^{-3}} = 8\times10^{-6} - 5\times10^{-11}$

$= 7.99995\cancel{5}\times10^{-6}$

$R_T = R_1 + R_2 + R_3 + R_4 + R_5$

$= 2.193\times10^{-3} + 43.86\times10^{-6} + 0.3508 + 43.86\times \ldots + 2.193\times \ldots$

$R_T = 0.3553\ \Omega$



$R = \frac{x}{2\sigma S(x)}$

$dR_2 = \frac{dx}{2\sigma S(x)}$

$S(x) = y(x)\,t$

$y(x) = 0.002 - 0.05x$

$R = \int_0^{0.02} \frac{dx}{2\sigma[0.002 - 0.05x]t}$

$R_2 = 1.2\times10^{-3}$

$R_T = 8.572\times10^{-3}\ \Omega$