An-Najah National University; Engineering Faculty; Electrical Eng. Dept.
Electromagnetic Theory I (63251); Dr. Naser Abu-Zaid;

14-3-2009 | **First Exam** | 1:20 hr.

Student Name.................................Student No.............................*Section* ......

*Insert your solutions **only** in the indicated places*

Question One) Two point charges Q_1 and Q_2 located at $P_1(1,2,0)$ and $P_2(2,0,0)$, respectively. Find the relation between Q_1 and Q_2 such that the total force on a test charge at the point $P(-1,1,0)$ has no x-component. **(6 marks)**

the total force on test charge at that point $= 0 \Rightarrow F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_R$

$R_{1P} = (-1-1, 1-2, 0-0)$

$\Rightarrow -2\hat{a}_x - \hat{a}_y$

$|R_1| = \sqrt{1+1} = \sqrt{3} = 1.7$

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$$F_1 = \frac{Q Q_P \vec{R}_1}{4\pi\varepsilon_0 R_1^3} = \frac{Q_1 Q_P}{4\times\pi\times\varepsilon_0} \frac{-2\hat{a}_x - \hat{a}_y}{(1.7)^3} = \frac{Q_1 Q_P(-2a_x - a_y)}{5.430\times10^{-10}}$$

$R_2P = (-1-2, 1, 0) \Rightarrow -3\hat{a}_x + \hat{a}_y$

$|R_{2P}| = \sqrt{10} = 3.162$

$$F_2 = \frac{Q_2 Q_P \vec{R}_2}{4\pi\varepsilon_0 R_2^3} = \frac{Q_2 Q_P}{4\pi\varepsilon_0} \frac{-3\hat{a}_x + \hat{a}_y}{(3.16)^3} = \frac{Q_2 Q_P(-3a_x + a_y)}{3.48\times10^{-9}}$$

$\Rightarrow$ if the force $= 0 \Rightarrow$ the the force equals and opposit

$$\frac{Q_1 Q_P(-2\hat{a}_x - \hat{a}_y)}{5.43\times10^{-10}} = \frac{Q_2 Q_P(-3\hat{a}_x + \hat{a}_y)}{3.48\times10^{-9}}$$

$Q_1(3.48\times10^{-9})(-2\hat{a}_x - \hat{a}_y) = Q_2(5.43\times10^{-10})(-3\hat{a}_x + \hat{a}_y)$

$Q_1(-6.96\times10^{-9}\hat{a}_x - 3.48\times10^{-9}\hat{a}_y) = Q_2(-16.29\times10^{-10}\hat{a}_x + 5.43 \ldots$

$Q_1 = ?\, Q_2$

Question Two) Given the electric flux density $\mathbf{D} = \frac{16}{r}\cos(2\theta)\,\hat{\mathbf{a}}_\theta$:

a) Use one side of the divergence theorem to find the total charge within the region $1 < r < 2$, $0.5 < \theta < 1\,(rad)$, $1.5 < \varphi < 2\,(rad)$.

b) Use the other side of the divergence theorem to find the total charge within the region $1 < r < 2$, $1 < \theta < 2\,(rad)$, $1 < \varphi < 2\,(rad)$. **(8 marks)**

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a) $\int \nabla \cdot D \, dV = Q_{enc} = \oint \vec{D} \cdot d\vec{s}$

$\nabla \cdot D$ = using spherical

$= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \theta}(D_\theta \sin\theta) + \frac{1}{r\sin\theta}\left(\frac{\partial D_\varphi}{\partial \varphi}\right)$

$= \frac{1}{r^2}\frac{\partial}{\partial r}(0) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \theta}\left(\frac{16}{r}\cos(2\theta)\sin\theta\right) + 0$

$= \frac{1}{r\sin(\theta)} \cdot \frac{\partial (16\cos(2\theta)\sin\theta)}{\partial \theta} = \frac{1}{r\sin\theta} \cdot 16[\cos(2\theta)\cos\theta + \sin\theta \cdot -\sin(2\theta)]$

$= \frac{1}{r\sin\theta}[16\cos(2\theta)\cos\theta - 32\sin\theta\sin(2\theta)]$

$= \left[\frac{16\cos(2\theta)\cos\theta}{r\sin\theta}\right] - \left[\frac{32\sin(2\theta)}{r}\right]$

$dv = dr \cdot r\,d\theta \cdot r\,d\varphi \sin\varphi$

$\int_{\varphi}\int_{\theta}\int_{r} \frac{16\cos(2\theta)\cos\theta}{r\sin\theta} \times r^2\sin\theta - 32\sin(2\theta)\,r\sin\theta \ldots$

$\int\int\int 16\,\frac{r\cos(2\theta)\cos\theta\sin\varphi}{\sin\theta}\,dr\,d\theta\,d\varphi - \int\int\int 32\,r\sin(2\theta)\sin\varphi\,dr\,d\ldots$

$\Rightarrow$ solve integral to find Q_{enc} = ?

b) $ds = d\varphi\, r\sin\varphi\, dr$

$\vec{D} = \frac{16}{r}\cos(2\theta)\,\hat{a}_\theta$

$\int_{1}^{2}\int_{1}^{2}\int_{1}^{2} \frac{16}{r}\cos(2\theta) \cdot r\sin\varphi\, dr\, d\theta\, d\varphi$ + another ds

$\int\int\int 16\cos(2\theta)\sin\varphi \cdot dr\,d\theta\,d\varphi$

ds

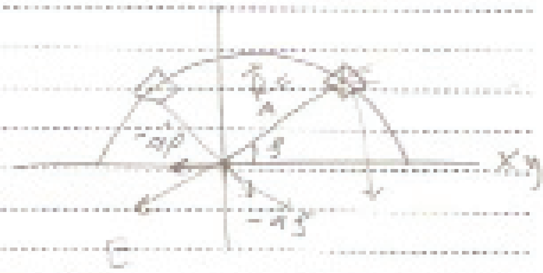
ds

cont

Question Three) A wire is bent in the form of a half loop (half circle in the upper xy plane) of radius a and charged with uniform line charge density $\rho_l \left(C/m\right)$. Calculate the electric field intensity at the center of the loop (at the origin) **(6 marks)**

$E = ??$

$$\int \frac{\rho_L \, dl \, \vec{R}}{4\pi \epsilon_0 R^3}$$



$R = \vec{r} - \vec{r'} \Rightarrow \vec{r} = 0$

$R = -\vec{r'} = -a\hat{a}_\rho + a\hat{a}_\phi$

$|R| = a$

$dl = a\, d\phi$

$$\int \frac{\rho_L \, a \, d\phi}{4\pi \epsilon_0 a^3} \left(-a\hat{a}_\rho - a\hat{a}_\phi\right)$$

$$\int \frac{\rho_L \, d\phi}{4\pi \epsilon_0 a^2} \left(-\hat{a}_\rho - \hat{a}_\phi\right) \Rightarrow \text{the } \hat{\rho} \text{ component canceleh}$$

$$\boxed{-\int \frac{\rho_L \, d\phi}{4\pi \epsilon_0 a^2} \hat{a}_\phi}$$

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Good Luck