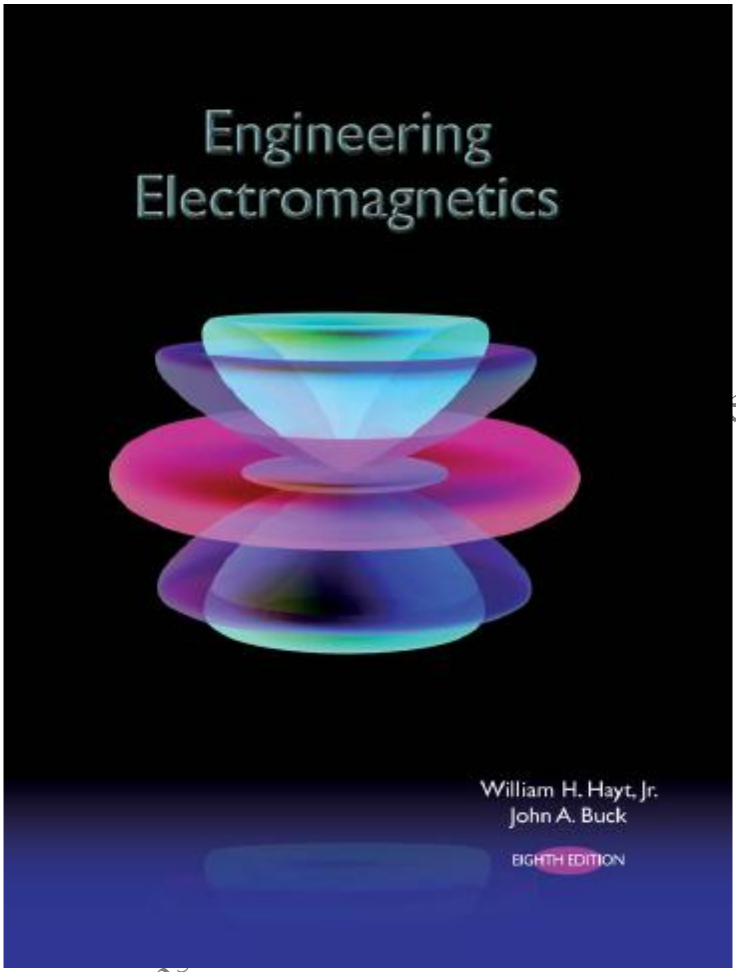
Text Book


Engineering
Electromagnetics
William H. Hayt, Jr.
John A. Buck
EIGHTH EDITION

Course syllabus

<table>
<tr><td>Electromagnetic 2 (63374)</td><td colspan="3">ELECTRICAL ENGINEERING</td></tr>
<tr><td>Semester</td><td colspan="3">5th & 6th</td></tr>
<tr><td>Language</td><td colspan="3">English</td></tr>
<tr><td>Compulsory / Elective</td><td colspan="3">Compulsory course for electrical engineering and communication engineering students.</td></tr>
<tr><td>Prerequisites</td><td colspan="3">Electromagnetic theory (1) and preferred after a course on differential equations.</td></tr>
<tr><td>Course Contents</td><td colspan="3">Magnetic Forces; Magnetic Circuits; Inductance; Faraday's Law; Displacement Current and time varying Maxwell's equations; Transmission lines; Plane electromagnetic waves; Reflection and transmission of plane EM waves; Introduction to waveguides.</td></tr>
<tr><td>Course Objectives</td><td colspan="3">To understand Faraday's Law and its applications; To analyze guided propagation through Transmission lines, and solve associated problems; To predict the existence of EM waves for time varying Maxwell's equations; To understand EM wave propagation in unbounded media and study the characteristics and parameters of plane EM waves and solutions of wave equations; To study the effect of dispersion in communication channels; To study basic applications of EM theory including TL's and waveguides; To be prepared for more advanced courses. To appreciate and feel the importance of electromagnetic theory in our daily life.</td></tr>
<tr><td rowspan="3">Learning Outcomes and Competences</td><td>1. Be able to apply knowledge of complex calculus, vector algebra and vector calculus to EM field problems (DC and time varying fields).</td><td>A</td><td>50%</td></tr>
<tr><td>2. Be able to analyze and design components and/or programs in relation to field problems.</td><td>C&K</td><td>10%</td></tr>
<tr><td>3. Attain the ability to solve basic electromagnetic wave propagation, reflection and refraction problems.</td><td>E</td><td>40%</td></tr>
<tr><td>Textbook and References</td><td colspan="3">1. "Engineering Electromagnetics", William H. Hayt and John A. Buck; 7th Edition; McGraw-Hill International Editions, 2006.
2. "Field and Wave Electromagnetics", David K. Cheng; Addison-Wesley Publishing Company; Second Edition 1989.
3. http://en.wikipedia.org/wiki/Electromagnetic_field</td></tr>
<tr><td>Assessment Criteria</td><td></td><td>If any,mark as (X)</td><td>Percent (%)</td></tr>
</table>

	Midterm Exams	X	40
	Quizzes	X	10
	Homework's		
	Projects		
	Term Paper		
	Laboratory Work		
	Other		
	Final Exam	X	50
Instructor(s)	Assist. Prof. Dr. Naser A. Abu-Zaid; naser_res@yahoo.com		
Week	**Subject**		
1-2	**Magnetic Forces:** Lorentz Force equation; Magnetic Forces and Torques; Magnetic materials and permeability; Magnetic Boundary conditions; Magnetic Circuits; Magneto-static energy; Inductance and Mutual inductance; Summary of Maxwell's equations for static and steady fields.		
3-4	**Time-Varying Fields and Maxwell's Equations:** Magnetic forces and torques; Magnetic materials and magnetic circuits;		
3-4	Faraday's Law and applications; Displacement current; Point form and Integral forms of Maxwell's equations; Electromagnetic Boundary Conditions;		
5	**Transmission Lines:** General Transmission Line Equations; TL Parameters; Lossless propagation; Lossless propagation of sinusoidal voltages; Complex analysis of sinusoidal waves; Solution of Transmission line equations in phasor form; lossless and low loss propagation; Power transmission and losses;		
	First Exam		
6	Wave reflections; VSWR; Finite length TL; TL's as circuit elements; Smith Chart; Transient Analysis(possible);		
7	**Uniform Plane Electromagnetic Waves:** Wave equations and their solutions; Propagation in free space; Propagation in dielectrics; propagation constant; intrinsic impedance; phase velocity, phase constant; attenuation constant, wave length;		
8	Flow of electromagnetic power and Poynting's Vector (Poynting's Thm.); Propagation in good conductors; Skin effect; Polarization of waves;		
9-10	**Reflection and Dispersion:** Normal Incidence at a Plane Dielectric Boundary; Normal Incidence at Plane Conducting Boundary;		
11-12	SWR; Reflection from multiple interfaces; Propagation in arbitrary directions;		
	Second Exam		
13	Oblique Incidence at a Plane Dielectric Boundary (Perpendicular		

	Polarization and Parallel Polarization); Oblique Incidence at Plane Conducting Boundary;
14	Total Reflection and Total Transmission; Dispersion and Pulse expansion; Introduction to metallic waveguides.
15	General Review

Magnetic Forces, Materials and Inductance

Lorentz force equation

The electric force on a particle whether its moving or stationary is

$$\mathbf{F}_e = Q\mathbf{E}$$

Positive charge implies force and field are in same direction, while negative charge implies opposite directions.

The magnetic force on a moving particle in a magnetic flux density $\mathbf{B}$ with velocity $\mathbf{v}$ is:

$$\mathbf{F}_m = Q\mathbf{v} \times \mathbf{B}$$

The total force is the superposition of both

$$\underbrace{F = \mathbf{F}_e + \mathbf{F}_m = Q[\mathbf{E} + \mathbf{v} \times \mathbf{B}]}_{\text{Lorentz force equation}}$$

Example 9.1: A point charge $Q = 18(nC)$, is moving with a velocity of $5 \times 10^6 (m/s)$ in a direction specified by $\hat{\mathbf{a}}_v = 0.6\hat{\mathbf{a}}_x + 0.75\hat{\mathbf{a}}_y + 0.3\hat{\mathbf{a}}_z$. Find the magnitude of the vector force exerted on the moving particle by the field:

1) $\mathbf{B} = -3\hat{\mathbf{a}}_x + 4\hat{\mathbf{a}}_y + 6\hat{\mathbf{a}}_z (mT)$.
2) $\mathbf{E} = -3\hat{\mathbf{a}}_x + 4\hat{\mathbf{a}}_y + 6\hat{\mathbf{a}}_z (KV/m)$.
3) Bothe fields acting together.

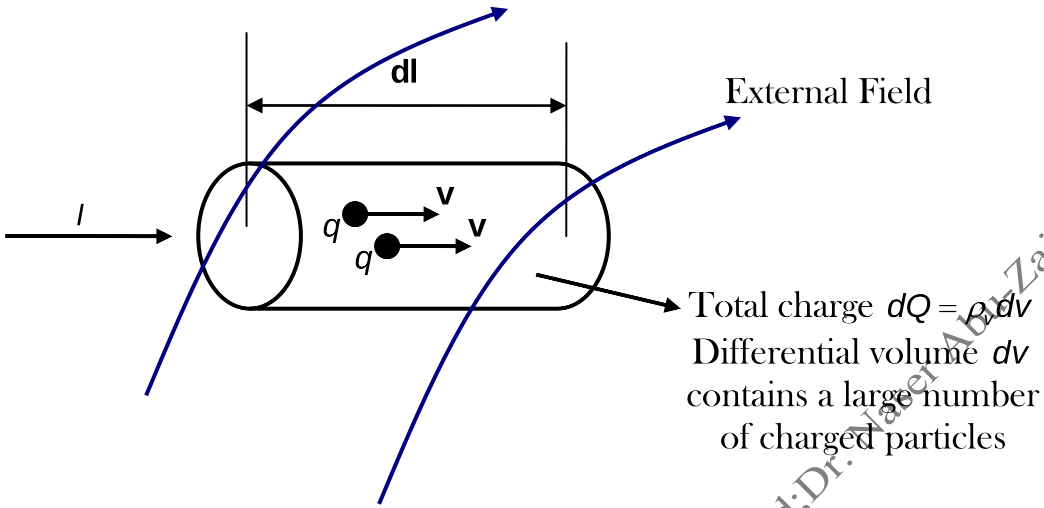
$$\boldsymbol{F}_m = Q\boldsymbol{v} \times \boldsymbol{B} = (18 \times 10^{-9})(5 \times 10^6)\left(0.6\hat{\boldsymbol{a}}_x + 0.75\hat{\boldsymbol{a}}_y + 0.3\hat{\boldsymbol{a}}_z\right) \times \left(-\mathbf{3}\hat{\boldsymbol{a}}_x + \mathbf{4}\hat{\boldsymbol{a}}_y + \mathbf{6}\hat{\boldsymbol{a}}_z\right) \cdot 10^{-3} = ?$$

$$\boldsymbol{F}_E = Q\boldsymbol{E} = (18 \times 10^{-9})\left(-\mathbf{3}\hat{\boldsymbol{a}}_x + \mathbf{4}\hat{\boldsymbol{a}}_y + \mathbf{6}\hat{\boldsymbol{a}}_z\right) \cdot 10^{-3} = ?$$

$$\boldsymbol{F} = \boldsymbol{F}_E + \boldsymbol{F}_m = ?$$

Force on a differential current element

Consider a small section of current:



$$I\mathbf{dl} = \mathbf{J}dv = \mathbf{K}ds$$

The force on each charged particle due to motion in a magnetic field is:

$$d\mathbf{F}_m = dQ\,\mathbf{v}\times\mathbf{B}$$

But

$$dQ = \rho_v dv$$

$$\Rightarrow d\mathbf{F}_m = \rho_v dv\,\mathbf{v}\times\mathbf{B}$$

And since

$$\mathbf{J} = \rho_v\,\mathbf{v}$$

Hence

$$d\mathbf{F}_m = \mathbf{J}\times\mathbf{B}dv = \mathbf{K}\times\mathbf{B}ds = I\mathbf{dl}\times\mathbf{B}$$

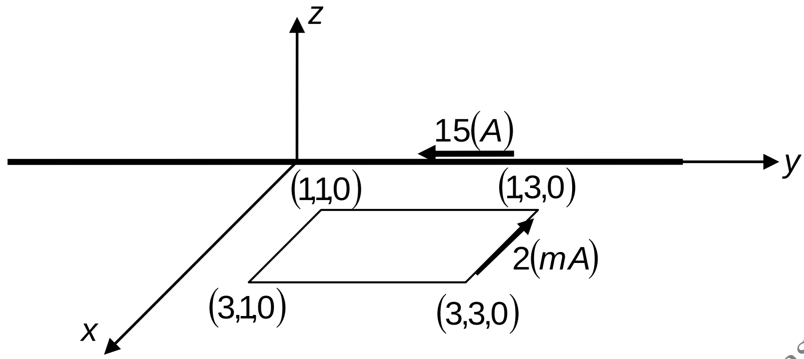
The total force on a conductor carrying current is:

$$\mathbf{F}_m = \iiint_v \mathbf{J}\times\mathbf{B}dv = \iint_s \mathbf{K}\times\mathbf{B}ds = \oint_c I\mathbf{dl}\times\mathbf{B} = -I\oint_c \mathbf{B}\times\mathbf{dl}$$

For uniform **B**

$$\mathbf{F}_m = I\mathbf{dl}\times\mathbf{B}$$

Example 9.2: In the figure shown. Find the net force on the closed loop due to the field produced by the straight filament.



$$H_{fil} = \frac{I_{fil}}{2\pi R}\hat{\boldsymbol{a}}_{\alpha} = \frac{\mathbf{2\times 10^{-3}}}{\mathbf{2\pi x}}\hat{\boldsymbol{a}}_{z}$$

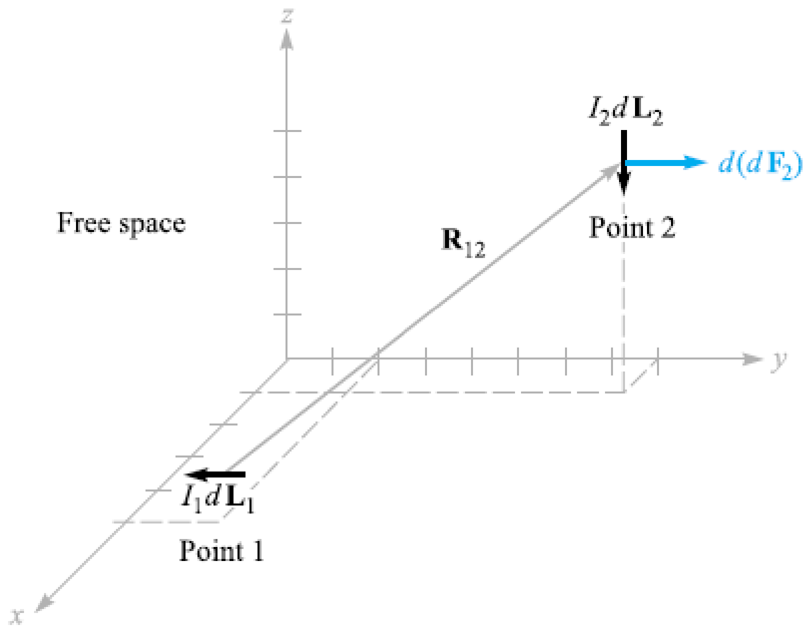
$$\boldsymbol{B}_{fil} = \mu_o \boldsymbol{H}_{fil} = \frac{\mathbf{30\times 10^{-7}}}{\boldsymbol{x}}\hat{\boldsymbol{a}}_{z}$$

$$\boldsymbol{F}_{loop} = -I_{loop}\oint_C \boldsymbol{B}_{fil}\times \boldsymbol{dl}$$

$$= (-2\times 10^{-3})(3\times 10^{-6})\left\{\begin{array}{l} \int_{x=1}^{3}\left(\frac{\hat{\boldsymbol{a}}_z}{x}\right)\times dx\hat{\boldsymbol{a}}_x + \int_{y=1}^{3}\left(\frac{\hat{\boldsymbol{a}}_z}{3}\right)\times dy\hat{\boldsymbol{a}}_y \\ + \int_{x=3}^{1}\left(\frac{\hat{\boldsymbol{a}}_z}{x}\right)\times dx\hat{\boldsymbol{a}}_x + \int_{y=3}^{1}\left(\frac{\hat{\boldsymbol{a}}_z}{1}\right)\times dy\hat{\boldsymbol{a}}_y \end{array}\right\}$$

$$= -8\hat{\boldsymbol{a}}_x (nN)$$

Force between differential current elements



The magnetic field at point 2 due to a current element at point 1 was found to be

$$d\mathbf{H}_2 = \frac{I_1 d\mathbf{L}_1 \times \mathbf{a}_{R12}}{4\pi R_{12}^2}$$

The differential force on a differential current element is

$$d\mathbf{F} = I\, d\mathbf{L} \times \mathbf{B}$$

letting $\boldsymbol{B}$ be $d\boldsymbol{B}_2$ (the differential flux density at point 2 caused by current element 1), by identifying $I\, d\boldsymbol{L}$ as $I_2 d\boldsymbol{L}_2$, and by symbolizing the differential amount of our differential force on element 2 as $d(d\boldsymbol{F}2)$:

$$d(d\mathbf{F}_2) = I_2 d\mathbf{L}_2 \times d\mathbf{B}_2$$

$$d\mathbf{B}_2 = \mu_0 d\mathbf{H}_2$$

$$d(d\mathbf{F}_2) = \mu_0 \frac{I_1 I_2}{4\pi R_{12}^2} d\mathbf{L}_2 \times (d\mathbf{L}_1 \times \mathbf{a}_{R12})$$

The total force between two filamentary circuits is obtained by integrating twice:

$$\mathbf{F}_2 = \mu_0 \frac{I_1 I_2}{4\pi} \oint \left[d\mathbf{L}_2 \times \oint \frac{d\mathbf{L}_1 \times \mathbf{a}_{R12}}{R_{12}^2} \right]$$

$$= \mu_0 \frac{I_1 I_2}{4\pi} \oint \left[\oint \frac{\mathbf{a}_{R12} \times d\mathbf{L}_1}{R_{12}^2} \right] \times d\mathbf{L}_2$$

EXAMPLE 8.2

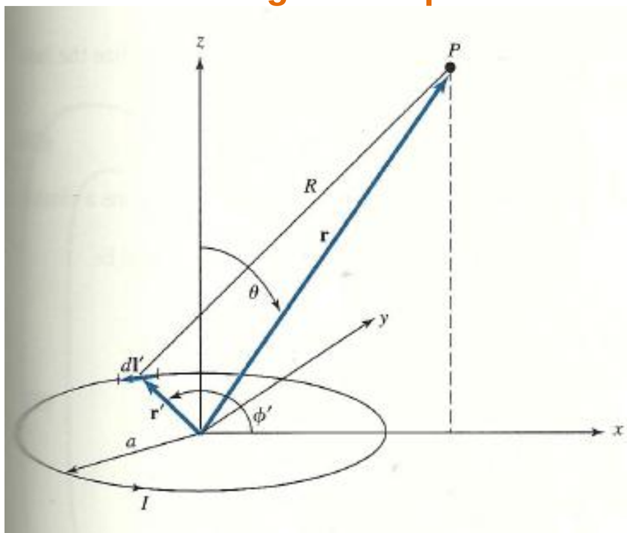
As an example that illustrates the use (and misuse) of these results, consider the two differential current elements shown in Figure 8.3. We seek the differential force on $d\mathbf{L}_2$.

Solution. We have $I_1 d\mathbf{L}_1 = -3\mathbf{a}_y \mathrm{A \cdot m}$ at $P_1(5, 2, 1)$, and $I_2 d\mathbf{L}_2 = -4\mathbf{a}_z \mathrm{A \cdot m}$ at $P_2(1, 8, 5)$. Thus, $\mathbf{R}_{12} = -4\mathbf{a}_x + 6\mathbf{a}_y + 4\mathbf{a}_z$, and we may substitute these data into (13),

$$d(d\mathbf{F}_2) = \frac{4\pi 10^{-7}}{4\pi} \frac{(-4\mathbf{a}_z) \times [(-3\mathbf{a}_y) \times (-4\mathbf{a}_x + 6\mathbf{a}_y + 4\mathbf{a}_z)]}{(16 + 36 + 16)^{1.5}}$$

$$= 8.56\mathbf{a}_y \text{ nN}$$

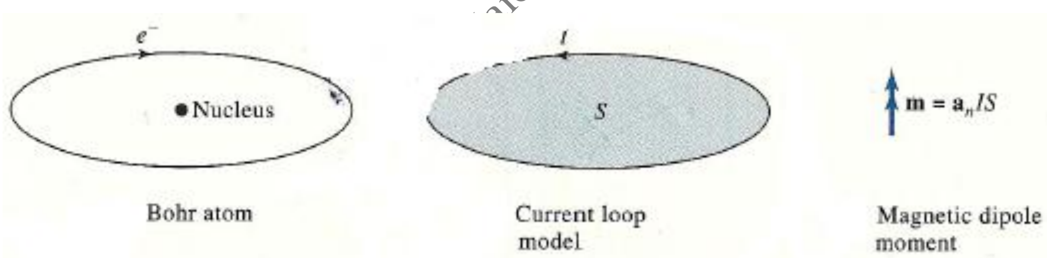
The Magnetic Dipole



Small loop of current I with radius a; First find $\boldsymbol{A}$ then $\boldsymbol{B}$. Using Binomial expansion with the assumption $r \gg a$, one can obtain;

$$\boldsymbol{A} \approx \widehat{\boldsymbol{a}}_{\varphi} \frac{\mu_o I(\pi a^2)\sin\theta}{4\pi r^2}$$

$$\boldsymbol{B} \approx \widehat{\boldsymbol{a}}_{r} \frac{\mu_o I(\pi a^2)\cos\theta}{2\pi 3} + \widehat{\boldsymbol{a}}_{\theta} \frac{\mu_o I(\pi a^2)\sin\theta}{4\pi r^3}$$



Define the magnetic dipole moment as;

$$\boldsymbol{m} = \widehat{\boldsymbol{a}}_n I\pi a^2 = \widehat{\boldsymbol{a}}_n IS = \widehat{\boldsymbol{a}}_n m \text{ (RHR)}$$

Let the center of the loop be located at r', then, the previously written expressions maybe written as;

$$\boldsymbol{A} \approx \frac{\mu_o \boldsymbol{m} \times \widehat{\boldsymbol{a}}_R}{4\pi R^2}$$

$$\boldsymbol{B} \approx \frac{\mu_o m}{4\pi r^3}(\boldsymbol{a_r}\, 2\cos\theta + \boldsymbol{a_\theta} \sin\theta\ )$$

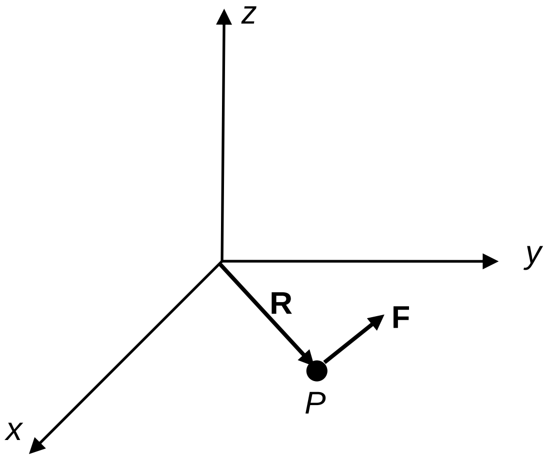
Compare with electric dipole

$$\boldsymbol{V} \approx \frac{\boldsymbol{p}.\widehat{\boldsymbol{a}}_R}{4\pi\varepsilon_o R^2}$$

$$\boldsymbol{E} \approx \frac{p}{4\pi\mu_o r^3}(\boldsymbol{a_r}\, 2\cos\theta + \boldsymbol{a_\theta} \sin\theta\ )$$

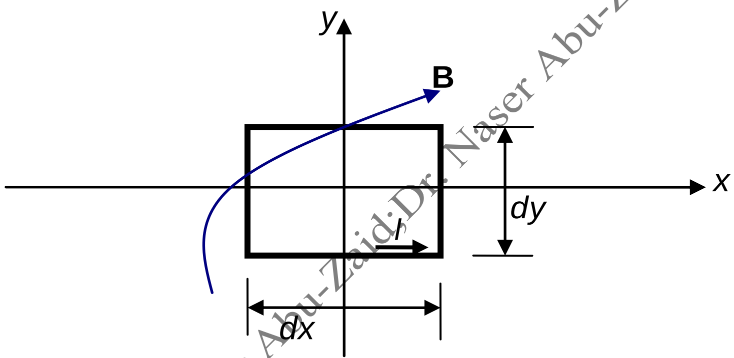
Forces and Torques on closed circuits

The torque $\mathbf{T}$ is defined as: $\mathbf{T} = \mathbf{R} \times \mathbf{F}$



- $\mathbf{T}$ is ⊥ to plane containing $\mathbf{R}$ and $\mathbf{F}$.
- The origin about which Torque is to be calculated must be defined.
- The point at which force is applied must also be defined.

Consider a differential current loop in an applied $\mathbf{B}$:



$$d\mathbf{T} = I \underbrace{d\mathbf{S}}_{\text{Differential vector area}} \times \mathbf{B}$$

Defining the differential magnetic dipole moment as;

$$d\mathbf{m} = I d\mathbf{S}$$

Then

$$d\mathbf{T} = d\mathbf{m} \times \mathbf{B}$$

So, the torque on a planar loop of any size or shape in a uniform $\mathbf{B}$ is:

$$\mathbf{T} = I\mathbf{S} \times \mathbf{B} = \mathbf{m} \times \mathbf{B}$$

The applied $\mathbf{B}$ would produce a torque which tends to turn the loop so as to align the magnetic field produced by the loop with the applied magnetic field.

EXAMPLE 8.3

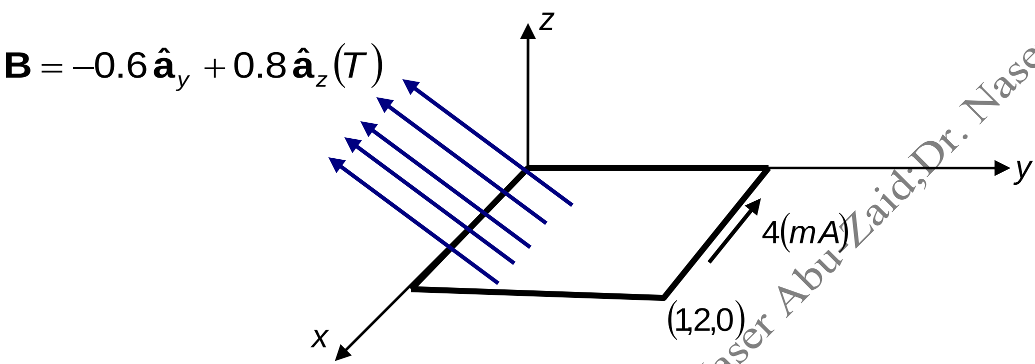
To illustrate some force and torque calculations, consider the rectangular loop shown in Figure 8.7. Calculate the torque by using $\mathbf{T} = I\mathbf{S} \times \mathbf{B}$.

Solution. The loop has dimensions of 1 m by 2 m and lies in the uniform field $\mathbf{B}_0 = -0.6\mathbf{a}_y + 0.8\mathbf{a}_z$T. The loop current is 4 mA, a value that is sufficiently small to avoid causing any magnetic field that might affect $\mathbf{B}_0$.

We have

$$\mathbf{T} = 4 \times 10^{-3}[(1)(2)\mathbf{a}_z] \times (-0.6\mathbf{a}_y + 0.8\mathbf{a}_z) = 4.8\mathbf{a}_x \text{ mN} \cdot \text{m}$$

Thus, the loop tends to rotate about an axis parallel to the positive x axis. The small magnetic field produced by the 4 mA loop current tends to line up with $\mathbf{B}_0$.



EXAMPLE 8.4

Now let us find the torque once more, this time by calculating the total force and torque contribution for each side.

Solution. On side 1 we have

$$\mathbf{F}_1 = I\mathbf{L}_1 \times \mathbf{B}_0 = 4 \times 10^{-3}(1\mathbf{a}_x) \times (-0.6\mathbf{a}_y + 0.8\mathbf{a}_z)$$
$$= -3.2\mathbf{a}_y - 2.4\mathbf{a}_z \text{ mN}$$

On side 3 we obtain the negative of this result,

$$\mathbf{F}_3 = 3.2\mathbf{a}_y + 2.4\mathbf{a}_z \text{ mN}$$

Next, we attack side 2:

$$\mathbf{F}_2 = I\mathbf{L}_2 \times \mathbf{B}_0 = 4 \times 10^{-3}(2\mathbf{a}_y) \times (-0.6\mathbf{a}_y + 0.8\mathbf{a}_z)$$
$$= 6.4\mathbf{a}_x \text{ mN}$$

with side 4 again providing the negative of this result,

$$\mathbf{F}_4 = -6.4\mathbf{a}_x \text{ mN}$$

Because these forces are distributed uniformly along each of the sides, we treat each force as if it were applied at the center of the side. The origin for the torque may be established anywhere since the sum of the forces is zero, and we choose the center of the loop. Thus,

$$\mathbf{T} = \mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_3 + \mathbf{T}_4 = \mathbf{R}_1 \times \mathbf{F}_1 + \mathbf{R}_2 \times \mathbf{F}_2 + \mathbf{R}_3 \times \mathbf{F}_3 + \mathbf{R}_4 \times \mathbf{F}_4$$
$$= (-1\mathbf{a}_y) \times (-3.2\mathbf{a}_y - 2.4\mathbf{a}_z) + (0.5\mathbf{a}_x) \times (6.4\mathbf{a}_x)$$
$$+ (1\mathbf{a}_y) \times (3.2\mathbf{a}_y + 2.4\mathbf{a}_z) + (-0.5\mathbf{a}_x) \times (-6.4\mathbf{a}_x)$$
$$= 2.4\mathbf{a}_x + 2.4\mathbf{a}_x = 4.8\mathbf{a}_x \text{ mN} \cdot \text{m}$$

Magnetization and Permeability

In free space

$$\mathbf{B} = \mu_0 \mathbf{H}$$

In material media, the *magnetization **M*** **defined** as the *magnetic dipole moment per unit volume,*

$$\mathbf{M} = \lim_{\Delta v \to 0} \frac{1}{\Delta v} \sum_{i=1}^{n \Delta v} \mathbf{m}_i$$

Its units must be the same as for **H**, amperes per meter.

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$$

for

linear isotropic media where a magnetic susceptibility χ_m can be defined:

$$\mathbf{M} = \chi_m \mathbf{H}$$

$$\mathbf{B} = \mu_0(\mathbf{H} + \chi_m \mathbf{H}) = \mu_0 \mu_r \mathbf{H}$$

$$\mu_r = 1 + \chi_m$$

$$\mu = \mu_0 \mu_r$$

$$\mathbf{B} = \mu \mathbf{H}$$

Table 8.1 Characteristics of magnetic materials

Classification	Magnetic Moments	B Values	Comments
Diamagnetic	$\mathbf{m}_{orb} + \mathbf{m}_{spin} = 0$	$B_{int} < B_{appl}$	$B_{int} \doteq B_{appl}$
Paramagnetic	$\mathbf{m}_{orb} + \mathbf{m}_{spin} =$ small	$B_{int} > B_{appl}$	$B_{int} \doteq B_{appl}$
Ferromagnetic	$\lvert\mathbf{m}_{spin}\rvert \gg \lvert\mathbf{m}_{orb}\rvert$	$B_{int} \gg B_{appl}$	Domains
Antiferromagnetic	$\lvert\mathbf{m}_{spin}\rvert \gg \lvert\mathbf{m}_{orb}\rvert$	$B_{int} \doteq B_{appl}$	Adjacent moments oppose
Ferrimagnetic	$\lvert\mathbf{m}_{spin}\rvert \gg \lvert\mathbf{m}_{orb}\rvert$	$B_{int} > B_{appl}$	Unequal adjacent moments oppose; low σ
Superparamagnetic	$\lvert\mathbf{m}_{spin}\rvert \gg \lvert\mathbf{m}_{orb}\rvert$	$B_{int} > B_{appl}$	Nonmagnetic matrix; recording tapes

EXAMPLE 8.5

Given a ferrite material that we shall specify to be operating in a linear mode with $B = 0.05$ T, let us assume $\mu_r = 50$, and calculate values for χ_m, M, and H.

Solution. Because $\mu_r = 1 + \chi_m$, we have

$$\chi_m = \mu_r - 1 = 49$$

Also,

$$B = \mu_r \mu_0 H$$

and

$$H = \frac{0.05}{50 \times 4\pi \times 10^{-7}} = 796 \text{ A/m}$$

The magnetization is $M = \chi_m H$, or 39,000 A/m. The alternate ways of relating B and H are, first,

$$B = \mu_0(H + M)$$

or

$$0.05 = 4\pi \times 10^{-7}(796 + 39,000)$$

showing that Amperian currents produce 49 times the magnetic field intensity that the free charges do; and second,

$$B = \mu_r \mu_0 H$$

or

$$0.05 = 50 \times 4\pi \times 10^{-7} \times 796$$

where we use a relative permeability of 50 and let this quantity account completely for the notion of the bound charges. We shall emphasize the latter interpretation in the chapters that follow.

Magnetic Boundary Conditions (DC)

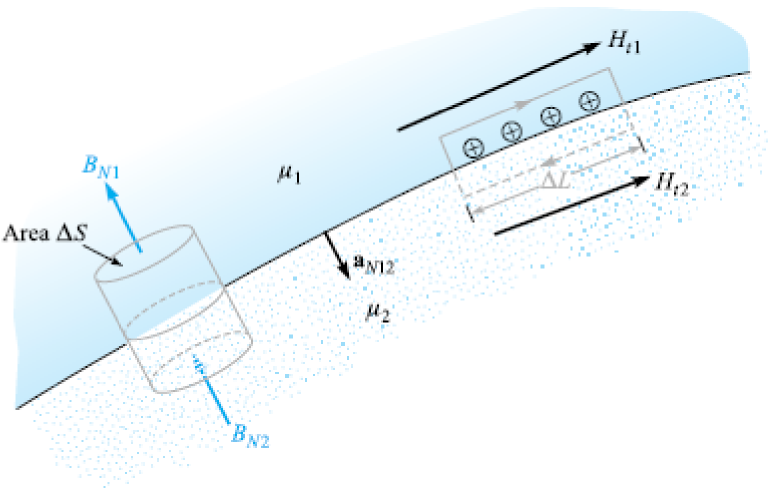


Figure 8.10 A gaussian surface and a closed path are constructed at the boundary between media 1 and 2, having permeabilities of μ_1 and μ_2, respectively. From this we determine the boundary conditions $B_{N1} = B_{N2}$ and $H_{t1} - H_{t2} = K$, the component of the surface current density directed into the page.

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

$$\boxed{B_{N2} = B_{N1}}$$

$$H_{N2} = \frac{\mu_1}{\mu_2} H_{N1}$$

$$\oint \mathbf{H} \cdot d\mathbf{L} = I$$

$$\boxed{H_{t1} - H_{t2} = K}$$

$$(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{a}_{N12} = \mathbf{K}$$

$$\mathbf{H}_{t1} - \mathbf{H}_{t2} = \mathbf{a}_{N12} \times \mathbf{K}$$

$$\frac{B_{t1}}{\mu_1} - \frac{B_{t2}}{\mu_2} = K$$

EXAMPLE 8.6

To illustrate these relationships with an example, let us assume that $\mu = \mu_1 = 4\,\mu$H/m in region 1 where $z > 0$, whereas $\mu_2 = 7\,\mu$H/m in region 2 wherever $z < 0$. Moreover, let $\mathbf{K} = 80\mathbf{a}_x$ A/m on the surface $z = 0$. We establish a field, $\mathbf{B}_1 = 2\mathbf{a}_x - 3\mathbf{a}_y + \mathbf{a}_z$ mT, in region 1 and seek the value of $\mathbf{B}_2$.

Solution. The normal component of $\mathbf{B}_1$ is

$$\mathbf{B}_{N1} = (\mathbf{B}_1 \cdot \mathbf{a}_{N12})\mathbf{a}_{N12} = [(2\mathbf{a}_x - 3\mathbf{a}_y + \mathbf{a}_z)\cdot(-\mathbf{a}_z)](-\mathbf{a}_z) = \mathbf{a}_z \text{ mT}$$

Thus,

$$\mathbf{B}_{N2} = \mathbf{B}_{N1} = \mathbf{a}_z \text{ mT}$$

We next determine the tangential components:

$$\mathbf{B}_{t1} = \mathbf{B}_1 - \mathbf{B}_{N1} = 2\mathbf{a}_x - 3\mathbf{a}y \text{ mT}$$

and

$$\mathbf{H}_{t1} = \frac{\mathbf{B}_{t1}}{\mu_1} = \frac{(2\mathbf{a}_x - 3\mathbf{a}_y)10^{-3}}{4 \times 10^{-6}} = 500\mathbf{a}_x - 750\mathbf{a}_y \text{ A/m}$$

Thus,

$$\begin{aligned}\mathbf{H}_{t2} &= \mathbf{H}_{t1} - \mathbf{a}_{N12} \times \mathbf{K} = 500\mathbf{a}_x - 750\mathbf{a}_y - (-\mathbf{a}_z) \times 80\mathbf{a}_x \\ &= 500\mathbf{a}_x - 750\mathbf{a}_y + 80\mathbf{a}_y = 500\mathbf{a}_x - 670\mathbf{a}_y \text{ A/m}\end{aligned}$$

and

$$\mathbf{B}_{t2} = \mu_2\mathbf{H}_{t2} = 7 \times 10^{-6}(500\mathbf{a}_x - 670\mathbf{a}_y) = 3.5\mathbf{a}_x - 4.69\mathbf{a}_y \text{ mT}$$

Therefore,

$$\mathbf{B}_2 = \mathbf{B}_{N2} + \mathbf{B}_{t2} = 3.5\mathbf{a}_x - 4.69\mathbf{a}_y + \mathbf{a}_z \text{ mT}$$

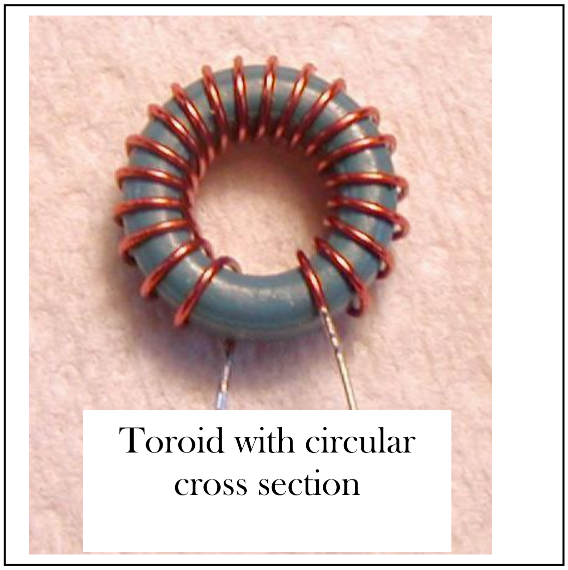
Magnetic Circuits and Hysteresis

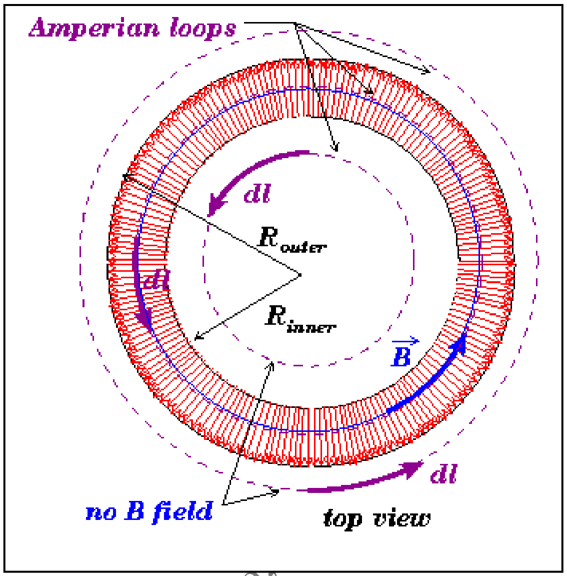
Magnetic Circuits Analogy with Electric Circuits

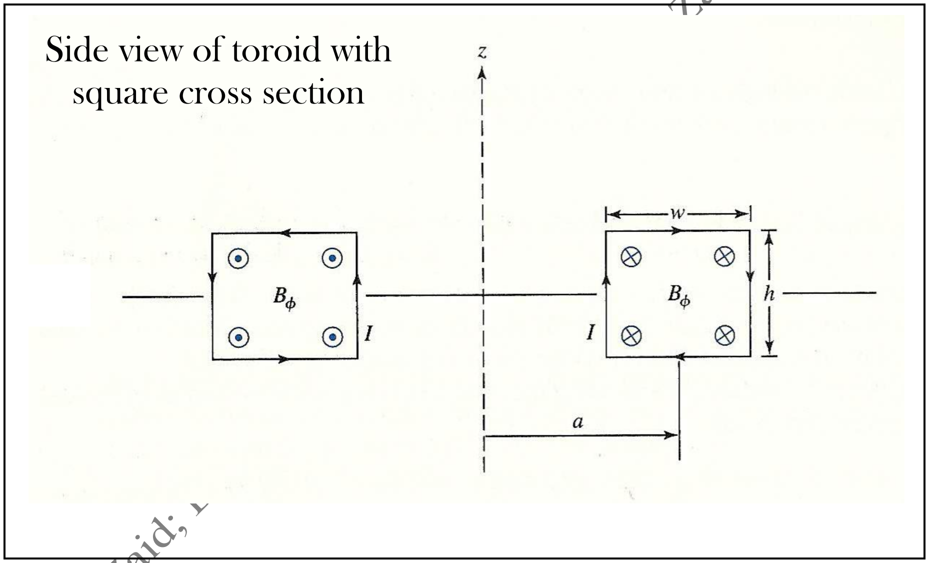
Electrostatics	Magnetostatics
$-\int_{\substack{chosen\\path}} \mathbf{E}\bullet d\mathbf{l} = V$	$\oint_C \mathbf{H}\bullet d\mathbf{l} = NI = V_m$
$\iint_S \mathbf{J}\bullet d\mathbf{s} = I$	$\iint_S \mathbf{B}\bullet d\mathbf{s} = \Phi_m$
$emf = V$	$mmf = V_m$
I	Φ_m
Resistance $R = \dfrac{d}{\sigma S}$	Reluctance $R_m = \dfrac{d}{\mu S}$
$V = RI$	$V_m = R_m \Phi_m$
σ	μ

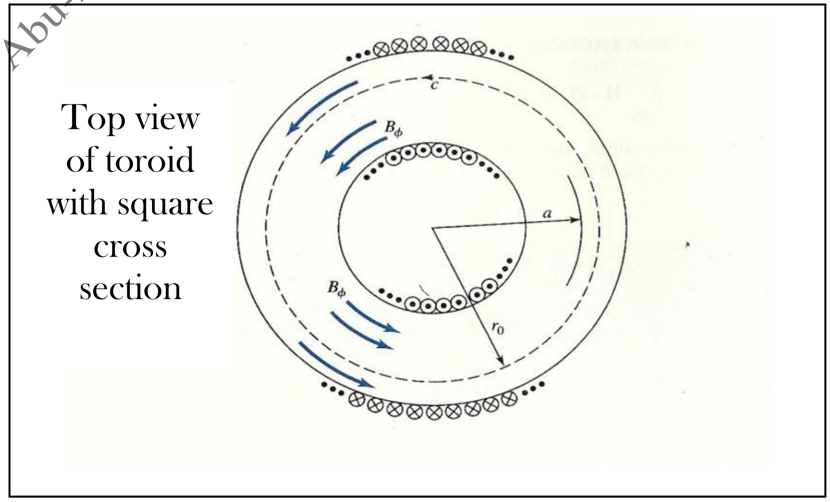
Obtaining the previous analogy

Consider a toroid with N turns, square cross-cross sectional area W^2, mean radius ρ_o and a steady coil current I.


Toroid with circular cross section


Amperian loops
dl
R outer
R inner
dl
B
dl
no B field
top view


Side view of toroid with square cross section
z
w
B_ϕ
B_ϕ
h
I
I
a


Top view of toroid with square cross section
c
B_ϕ
a
B_ϕ
r_0

Ampers law implies:

$$\oint_C \mathbf{H} \bullet \mathbf{dl} = NI = V_m$$

$$\Rightarrow H_\varphi = \frac{NI}{2\pi\rho_o}$$

$$\Rightarrow B_\varphi = \frac{\mu NI}{2\pi\rho_o}$$

Then:

$$\iint_S \mathbf{B} \bullet \mathbf{ds} = \Phi_m$$

$$\Rightarrow \Phi_m = \frac{\mu NI}{2\pi\rho_o} W^2$$

Knowing that:

$$V_m = NI\left(A.t\right)$$

$$\frac{1}{R_m} = \frac{\mu}{2\pi\rho_o} W^2$$

So:

$$R_m = \frac{2\pi\rho_o}{\mu W^2} = \frac{d}{\mu S}\left(H^{-1}\right)$$

$$\Phi_m = \frac{V_m}{R_m}$$

But, its well known that:

$$I = \frac{V}{R}$$

And its clear that:

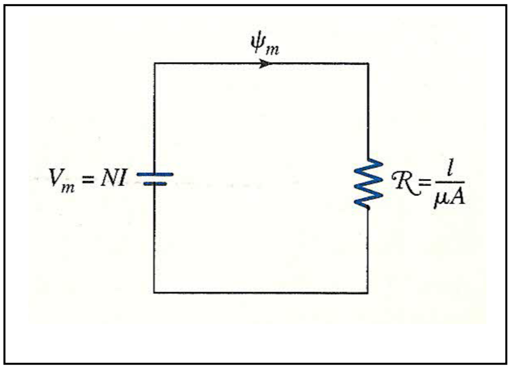
$$\Phi_m \rightarrow I$$

$$V_m \rightarrow V$$

$$\underbrace{R_m}_{\frac{d}{\mu S}} \rightarrow \underbrace{R}_{\frac{d}{\sigma S}}$$

$$\mu \rightarrow \sigma$$

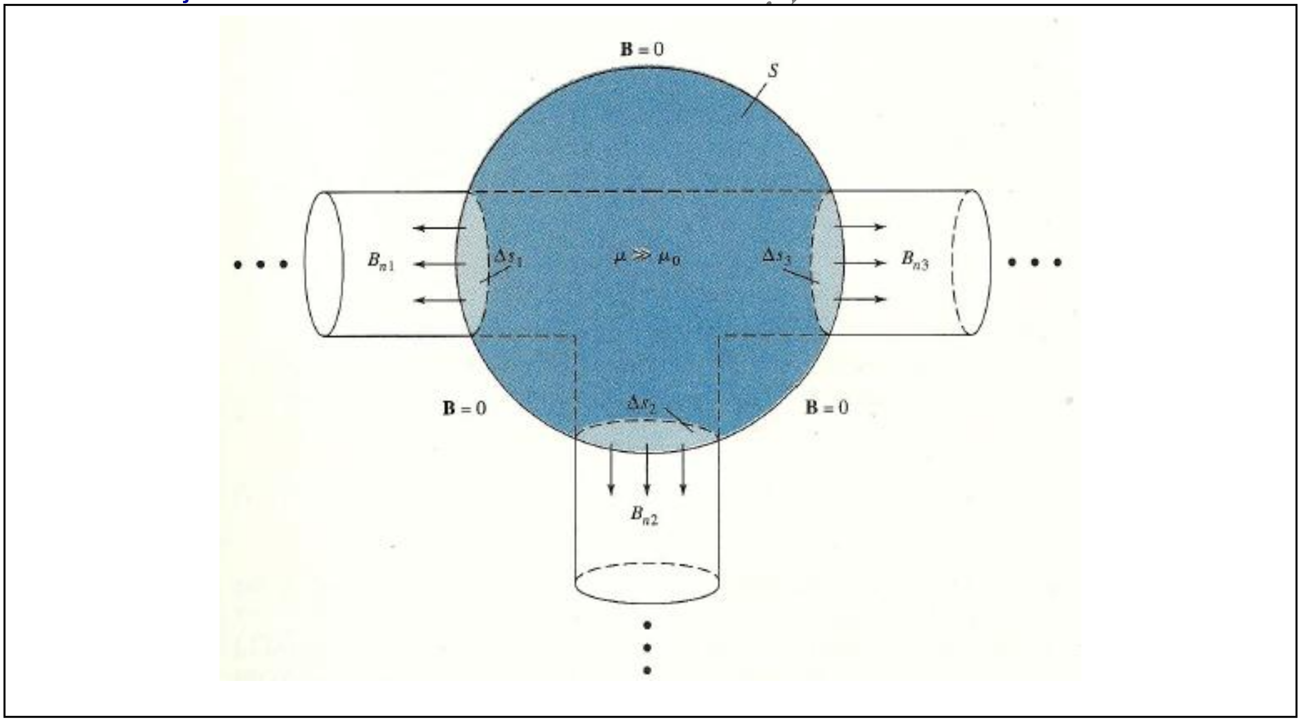
And the toroid circuit is replaced by the electric circuit given below:



Actually KVL for magnetic Circuits may by written as:

$$\underbrace{\sum_i V_{m_i}}_{\text{Sum of all mmf's from source coils}} = \underbrace{\sum_j R_{m_j} \Phi_{m_j}}_{\text{Sum of all mmf drops along the core}}$$

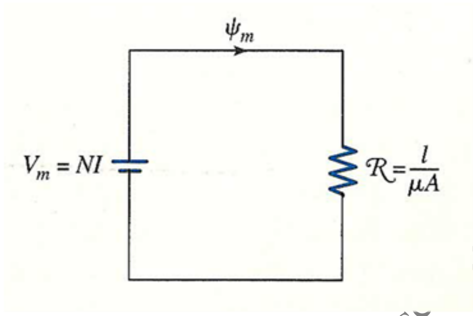
Similarly, KCL for magnetic circuits may be obtained by considering the boundary condition across a core junction:



$$B_{n1}S_1 + B_{n2}S_2 + B_{n3}S_3 = 0$$

$$\sum_i \Phi_{m_i} = 0$$

Example 4: Air core toroid with 500 turns, circular cross-section of $6cm^2$, mean radius of $15cm$, and coil current of 4A. Find H.



Solution:

If Ampers law is used:

$$H_\varphi = \frac{NI}{2\pi\rho_o} = \frac{500 \times 4}{6.28 \times 0.15} = 2120$$

Using electrical circuit analogy, then toroid circuit is replaced by the electric circuit given above:

$$R_m = \frac{d}{\mu S} = \frac{2\pi \times 0.15}{4\pi \times 10^{-7} \times 6 \times 10^{-4}}$$

$$= 1.25 \times 10^{-9} \left(H^{-1}\right)$$

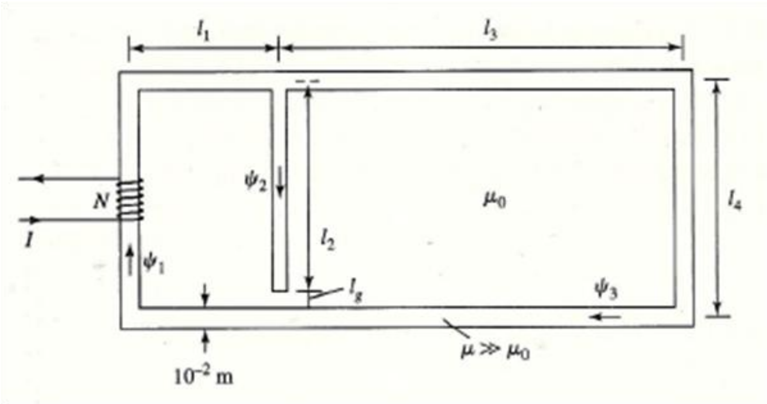
$$\Phi_m = \frac{V_{m_{source}}}{R_m} = \frac{2000}{1.25 \times 10^9}$$

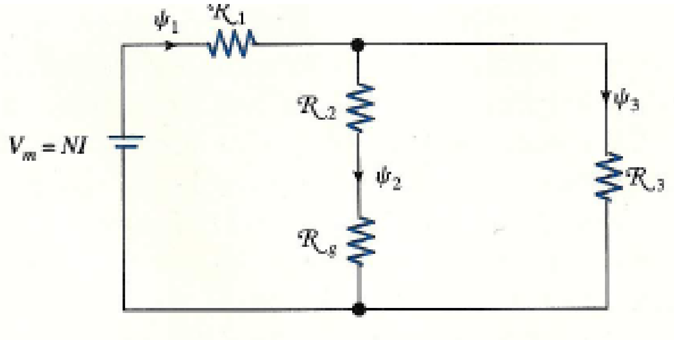
$$= 1.6 \times 10^{-6} \left(Wb\right)$$

$$B = \frac{\Phi_m}{S} = \frac{1.6 \times 10^{-6}}{6 \times 10^{-4}} = 2.67 \times 10^{-3} \left(T\right)$$

$$H = \frac{B}{\mu_o} = \frac{2.67 \times 10^{-3}}{4\pi \times 10^{-7}} = 2120 \left(A.t/m\right)$$

Example 5: Determine the magnetic flux through the airgap. $\mu_r = 1000$, $S = 10^{-4} m^2$ everywhere, $l_1 = 0.1m$, $l_g = 0.1mm$, $l_3 = 0.3m$, and $l_4 = 0.2m$, The source has 1000 turns of wire, with $I = 1mA$.





Solution:

$$R_{m_1} = \frac{d}{\mu S} = \frac{2l_1 + l_4}{1000\mu_o S} = 3.183\times 10^6 \left(H^{-1}\right)$$

$$R_{m2} = \frac{d}{\mu S} = \frac{l_4 - l_g - \left(10^{-2}/2\right)}{1000\mu_o S} = 1.551\times 10^6 \left(H^{-1}\right)$$

$$R_{m3} = \frac{d}{\mu S} = \frac{2l_3 + l_4}{1000\mu_o S} = 6.366\times 10^6 \left(H^{-1}\right)$$

$$R_g = \frac{d}{\mu S} = \frac{l_g}{\mu_o S} = 795.8\times 10^6 \left(H^{-1}\right)$$

$$\Phi_1 = \frac{V_m}{R_{m1} + \left(R_{m2} + R_g\right) // R_{m3}} = 1.053\times 10^{-7}\left(Wb\right)$$

Flux division gives:

$$\Phi_2 = \frac{R_{m3}}{R_{m3} + R_{m2} + R_g}\Phi_1 = 8.34\times 10^{-10}\left(Wb\right)$$

Ferromagnetic materials (Revisited)

Magnetic Hysteresis

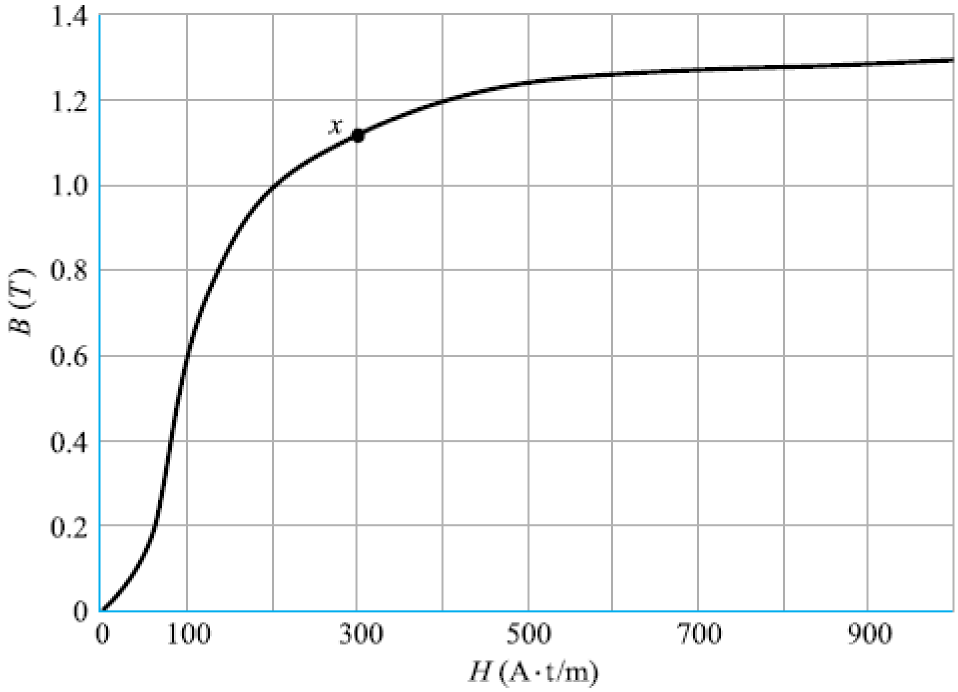


Figure 8.11 Magnetization curve of a sample of silicon sheet steel.

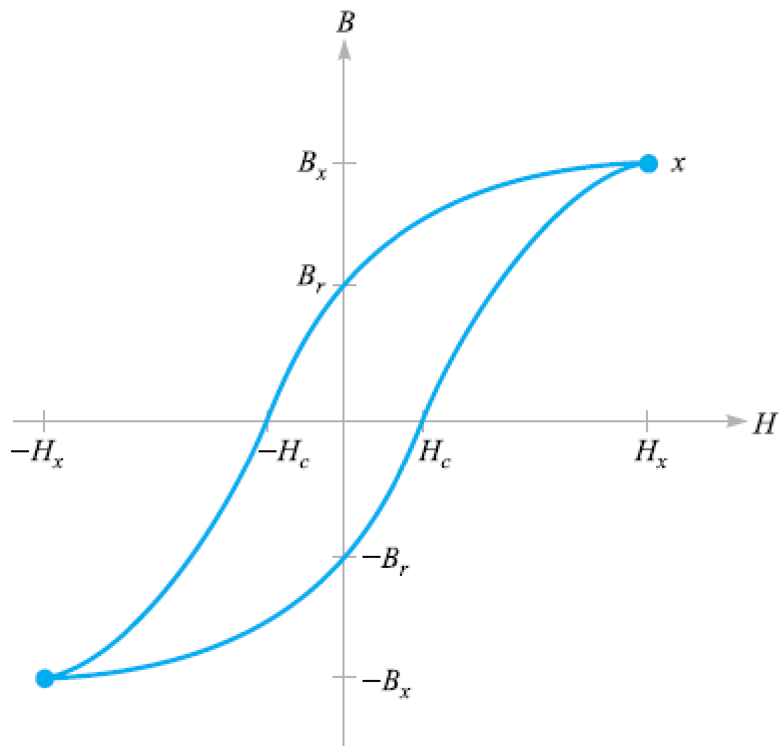


Figure 8.12 A hysteresis loop for silicon steel. The coercive force H_c and remnant flux density B_r are indicated.

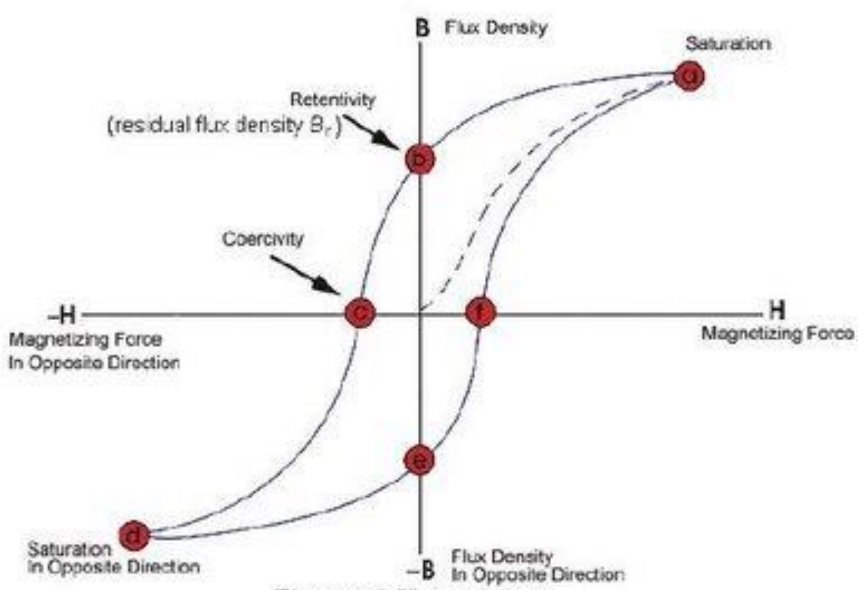


Figure 4.6: Hyteresis loop.

- Energy is lost.

- Easily magnetized and demagnetized. (Soft magnetic materials).
- (Hard to demagnetize once they are magnetized). Usefull for producing permenant magnets.
- Soft magnetic materials: Iron (0.2% impure), $\mu_{r_{max}} = 9000$, $H_c = 80 \left(A/m \right)$, $B_r = 0.77(T)$, saturation flux $B_s = 2.15(T)$
- Hard magnetic materials: Alinco (Aluminum-Nickel-Cobalt), $\mu_r = 3-5$, $H_c = 60 \left(KA/m \right)$, $B_r = 1.25(T)$, Curie Temp. $850(C^o)$

POTENTIAL ENERGY AND FORCES ON MAGNETIC MATERIALS

The total energy stored in a steady magnetic field in which $\boldsymbol{B}$ is linearly related to $\boldsymbol{H}$ is

$$W_H = \frac{1}{2} \iiint_v \boldsymbol{B} \cdot \boldsymbol{H} dv$$

Letting $\boldsymbol{B} = \mu \boldsymbol{H}$, we have the equivalent formulations

$$W_H = \frac{1}{2} \iiint_v \mu H^2 dv = \frac{1}{2} \iiint_v \frac{B^2}{\mu} dv$$

Think of this energy as being distributed throughout the volume with an energy density of

$$w_H = \frac{1}{2} \boldsymbol{B} \cdot \boldsymbol{H} \left(\frac{J}{m^3} \right)$$

To calculate the forces on nonlinear magnetic materials, suppose a long solenoid with a silicon-steel core. A coil containing $n\ turns/m$ with a current I surrounds it. Let the magnetic flux density be B_{st}. Suppose that the core is composed of two semi-infinite cylinders that are just touching.We now apply a mechanical force to separate these two sections of the core while keeping the flux density constant. We apply a force F over a distance dL, thus doing work $F\,dL$. We use the ***principle of virtual work*** to determine the work we have done in moving one core appearing as stored energy in the air gap we have created. This increase is

$$dW_H = FdL = \frac{1}{2} \frac{B_{st}^2}{\mu_o} SdL$$

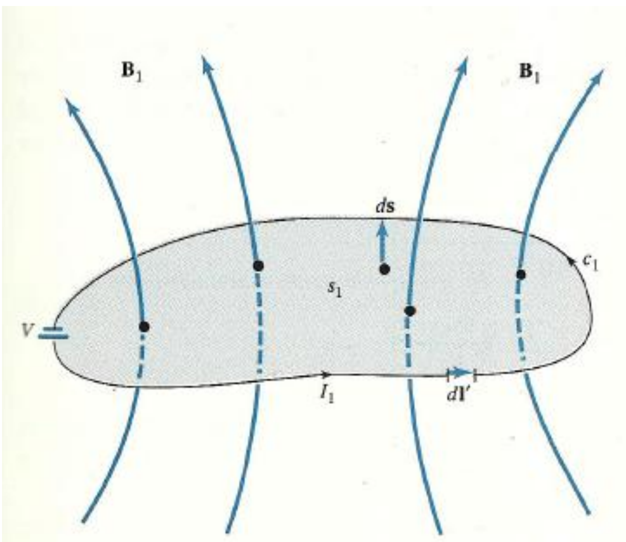
Where S is the core cross-sectional area. Thus

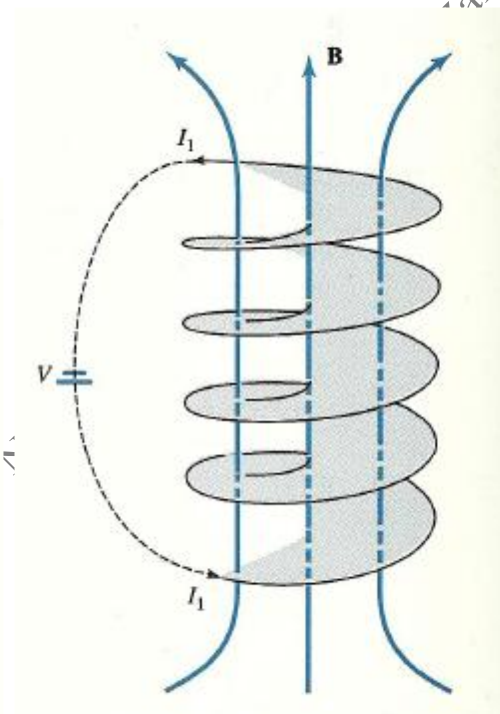
$$F = \frac{1}{2}\frac{B_{st}^2}{\mu_o}S$$

In summary, ***the principle of virtual work*** states that

$$\boldsymbol{F} = -\nabla W_H$$

INDUCTANCE AND MUTUAL INDUCTANCE





$$\Phi_{11} = \iint_{S_1} B_1 \cdot dS_1$$

$$B_1 = \frac{\mu I_1}{4\pi}\oint_{C_1}\frac{dl' \times \hat{a}_R}{R^2}$$

$$\Phi_{11} = \frac{\mu I_1}{4\pi}\iint_{S_1}\left(\oint_{C_1}\frac{dl' \times \hat{a}_R}{R^2}\right).dS_1$$

$$\Phi_{11} = L_{11}I_1$$

$$\Lambda_{11} = \mathrm{N}\Phi_{11}$$

Consider a coil of N turns in which a current I produces a total flux Φ. We assume that this flux links or encircles each of the N turns, and that each of the N turns links the total flux Φ.

The ***flux linkage*** $\Lambda = N\Phi$ *is defined as the product of the number of turns N and the flux Φ linking each of them*.

Definition of *inductance* (or self-inductance): the ratio of the total flux linkages to the current which they link;

$$L = \frac{\Lambda}{I} = \frac{N\Phi}{I}\left(H = \frac{Wb.t}{A}\right)$$

Illustration: To calculate the inductance per meter length of a coaxial cable of inner radius a and outer radius b. We may take the expression for total flux developed previously,

$$\Phi = \frac{\mu_0 I d}{2\pi} \ln\frac{b}{a}$$

and obtain the inductance rapidly for a length d,

$$L = \frac{\mu_0 d}{2\pi} \ln\frac{b}{a} \ \text{H}$$

or, on a per-meter basis,

$$L = \frac{\mu_0}{2\pi} \ln\frac{b}{a} \ \text{H/m}$$

In this case, $N = 1$ turn, and all the flux links all the current.

Illustration: For a toroidal coil of N turns and a current I, we have

$$B_\phi = \frac{\mu_0 N I}{2\pi\rho}$$

If the dimensions of the cross section are small compared with the mean radius of the toroid ρ_o, then the total flux is

$$\Phi = \frac{\mu_0 N I S}{2\pi\rho_0}$$

The inductance

$$L = \frac{\mu_0 N^2 S}{2\pi\rho_0}$$

Assumed that all the flux links all the turns.
A definition for inductance using energy expression,

$$L = \frac{2W_H}{I^2}$$

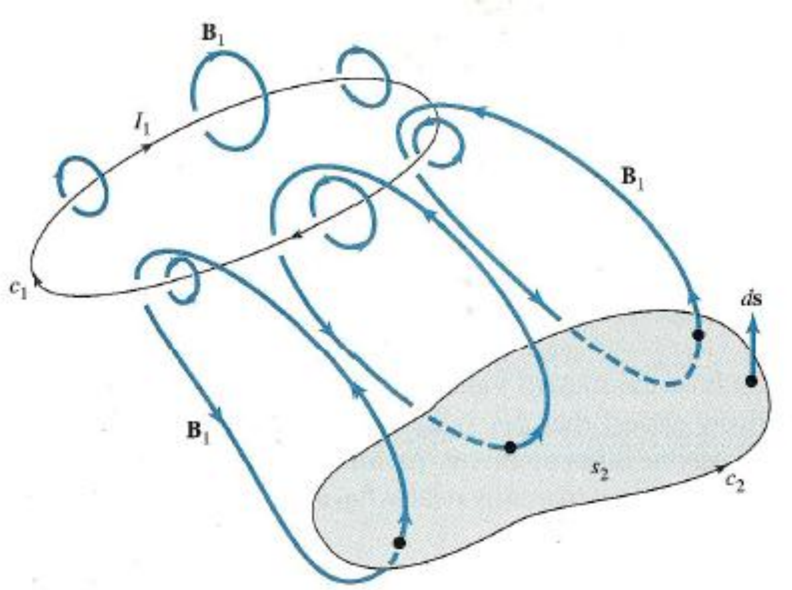
The interior of any conductor also contains magnetic flux. ***Internal inductance***, which must be combined with the external inductance to obtain the total inductance.

The internal inductance of a long, straight wire of circular cross section, radius *a*, and uniform current distribution is

$$L_{a,\text{int}} = \frac{\mu}{8\pi} \quad \text{H/m}$$

Try this one

Mutual inductance:



$$\Phi_{12} = \iint_{S_2} B_1 \cdot dS_2$$

$$B_1 = \frac{\mu I_1}{4\pi} \oint_{C_1} \frac{dl' \times \hat{a}_R}{R^2}$$

$$\Phi_{12} = \frac{\mu I_1}{4\pi} \iint_{S_2} \left(\oint_{C_1} \frac{dl' \times \hat{a}_R}{R^2} \right) . dS_2$$

$$\Phi_{12} = M_{12} I_1$$

$$\Lambda_{12} = N_2 \Phi_{12}$$

Mutual inductance between circuits 1 and 2, M_{12}, in terms of mutual flux linkages,

$$M_{12} = \frac{N_2 \Phi_{12}}{I_1}$$

where M_{12} signifies the flux produced by I_1 which links the path of the filamentary current I_2, and N_2 is the number of turns in circuit 2. It can be shown that

$$M_{12} = M_{21}$$

EXAMPLE 8.9

Calculate the self-inductances of and the mutual inductances between two coaxial solenoids of radius R_1 and R_2, $R_2 > R_1$, carrying currents I_1 and I_2 with n_1 and n_2 turns/m, respectively.

Solution. We first attack the mutual inductances. From Eq. (15), Chapter 7, we let $n_1 = N/d$, and obtain

$$\mathbf{H}_1 = n_1 I_1 \mathbf{a}_z \quad (0 < \rho < R_1)$$
$$= 0 \quad (\rho > R_1)$$

and

$$\mathbf{H}_2 = n_2 I_2 \mathbf{a}_z \quad (0 < \rho < R_2)$$
$$= 0 \quad (\rho > R_2)$$

Thus, for this uniform field

$$\Phi_{12} = \mu_0 n_1 I_1 \pi R_1^2$$

and

$$M_{12} = \mu_0 n_1 n_2 \pi R_1^2$$

Similarly,

$$\Phi_{21} = \mu_0 n_2 I_2 \pi R_1^2$$
$$M_{21} = \mu_0 n_1 n_2 \pi R_1^2 = M_{12}$$

If $n_1 = 50$ turns/cm, $n_2 = 80$ turns/cm, $R_1 = 2$ cm, and $R_2 = 3$ cm, then

$$M_{12} = M_{21} = 4\pi \times 10^{-7}(5000)(8000)\pi(0.02^2) = 63.2 \text{ mH/m}$$

The self-inductances are easily found. The flux produced in coil 1 by I_1 is

$$\Phi_{11} = \mu_0 n_1 I_1 \pi R_1^2$$

and thus

$$L_1 = \mu_0 n_1^2 S_1 d \quad \text{H}$$

The inductance per unit length is therefore

$$L_1 = \mu_0 n_1^2 S_1 \quad \text{H/m}$$

or

$$L_1 = 39.5 \quad \text{mH/m}$$

Similarly,

$$L_2 = \mu_0 n_2^2 S_2 = 22.7 \quad \text{mH/m}$$