An Najah University – Faculty of engineering
Electrical Eng. Department
System and Signals

[redacted]

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10.5 / 25

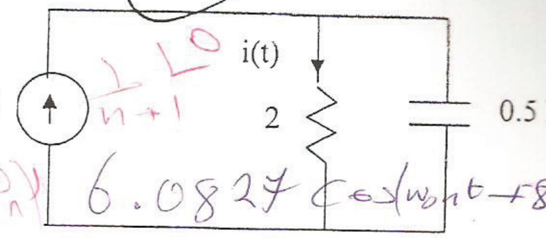
1st hr. Exam
summer 2009

Question #1 (10 mark)

Periodic signal has the following Fourier series

$$f(t)=\sum_{n=1}^{3}\frac{1}{n+1}\cos(nw_0t)+\sum_{n=1}^{3}\frac{2}{n}\sin(nw_0t)$$

$0 + C\,Q_6$



$\frac{1}{n+1}\angle 0$

$I(t) = \sum_{n=1}^{3} C_n \cos(nw_0t+\theta_n)$

$6.0827 \cos(w_0 n t + 80)$

Where $w_0 = 2$ rad/sec

Find: 1- sketch spectrum "magnitude and Phase" for C_n "complex coefficients" for the periodic signal

2- The signal applied to the circuit shown, find i(t)

3- What is the power dissipated on the resistance

$\frac{1}{n+1}(1) + \frac{2}{n}(j) * \left(\frac{\frac{1}{jwc}}{\frac{1}{jwc}+2}\right)$

$w_0 = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{w_0}, \quad T = \pi$

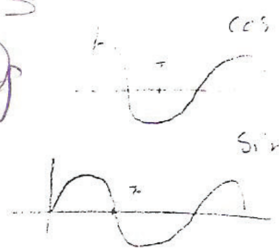
$C_n = \frac{1}{T}\int_0^T f(t)\, e^{-jnw_0t} = \frac{1}{\pi}\int_0^\pi \frac{1}{n+1}\cos(n2t)\, e^{-j2nt}\,dt + \frac{1}{\pi}\int_0^\pi \frac{2}{n}\sin(n2t)\, e^{-j2nt}\,dt$

let $u = \cos 2nt$ $\quad dv = e^{-j2nt}\,dt$

$du = \frac{-1}{2n}\sin 2nt\,dt$ $\quad v = \frac{1}{-j2n} e^{-j2nt}$

$\theta \angle \theta$

$8' \cos(w_0 n t + \theta)$



$\int_0^\pi \cos 2nt\, e^{-j2nt} = \cos 2nt * \frac{j}{2n} e^{-j2nt}\Big]_0^\pi + \frac{j}{4n^2}\int_0^\pi \sin 2nt\, e^{-j2nt}\,dt$

let $u = \sin 2nt$ $\quad dv = e^{-j2nt}\,dt$

$du = \frac{1}{2n}\cos 2nt$ $\quad v = \frac{j}{2n} e^{-j2nt}$

$\therefore \int_0^\pi \cos 2nt\, e^{-j2nt} = \frac{j}{2n}\cos 2n\pi\, e^{-jn2\pi} - \frac{j}{2n}\cos 0 + \cancel{\frac{j^2}{8n^3}\sin 2nt\, e^{-j2nt}\Big]_0^\pi}^{\,0} - \frac{j^2}{16n^4}\int_0^\pi \cos 2nt\, e^{-j2nt}\,dt$

$\frac{16n^4 + j^2}{16n^4}\int_0^\pi \cos 2nt\, e^{-j2nt}\,dt = \left[\frac{j}{2n}e^{-j2n\pi} - \frac{j}{2n}\right] \Rightarrow \int_0^\pi \cos 2nt\, e^{-j2nt}\,dt = \frac{j(e^{-j2n\pi}-1)}{2n} * \frac{16n^4}{16n^4+j^2} = \frac{j8n^3}{16n^2-1}(e^{-j2n\pi}-1)$

$\int_0^\pi \sin 2nt\, e^{-j2nt}\,dt =$ $\quad$ let $u = \sin 2nt$ $\quad dv = e^{-j2nt}\,dt$

$du = \frac{\cos 2nt}{2n}\,dt$ $\quad v = \frac{j}{2n} e^{-j2nt}$

$= \sin 2nt * \frac{j}{2n} e^{-j2nt}\Big]_0^\pi - \frac{j}{4n^2}\int \cos 2nt\, e^{-j2nt}\,dt$ $\quad$ let $u = \cos 2nt$ $\quad dv = e^{-j2nt}\,dt$

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$du = -\frac{\sin 2nt}{2n}$ $\quad v = \frac{j}{2n} e^{-j2nt}$

$= \frac{-j^2}{8n^3}\cos 2nt\, e^{-j2nt}\Big]_0^\pi - \frac{j^2}{16n^4}\int_0^\pi \sin 2nt\, e^{-j2nt}\,dt$

$\frac{16n^4 + j^2}{2n\,\cancel{16n^4}}\int_0^\pi \sin 2nt\, e^{-j2nt}\,dt = \frac{-j^2}{8n^3}e^{-j2n\pi} - \frac{j^2}{8n^3} = \frac{-j^2(e^{-j2n\pi}+1)}{\cancel{8n^3}}$

$\int_0^\pi \sin 2nt\, e^{-j2nt}\,dt = \frac{2n}{16n^4+j^2} * -j^2(e^{-j2n\pi}+1) = \frac{2n}{16n^2-1}(e^{-j2n\pi}+1)$

$C_n = \frac{1}{\pi(n+1)} * \frac{j8n^3}{16n^2-1}(e^{-j2n\pi}-1) + \cancel{\frac{4n}{\pi n(16n^2-1)}}(e^{-j2n\pi}+1) =$

$\frac{1}{\pi(16n^2-1)}\left[\frac{j8n^3}{n+1}(e^{-j2n\pi}-1) + 4e^{-j2n\pi} + 4\right]$

...stion # 2 (10 mark)

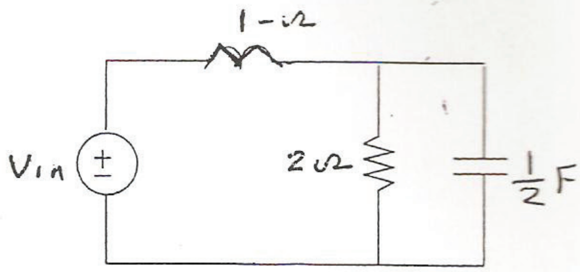
For the circuit shown:

If ~~Vin(t) = δ(t-1)~~ applied to the input, what is Vc(t)

$V_{in} = e^{-(t-1)} \mu(t-1)$

$V_c(j\omega) = \frac{2}{3} * \frac{1}{j\omega+1} e^{-j\omega}$

$V_c(t) = \frac{2}{3} e^{-(t-1)} \mu(t-1)$



b) for system shown, find and sketch i(t), knowing that Vin(t) given by the following signal

1- $Z(j\omega) = \frac{1}{j\omega}$

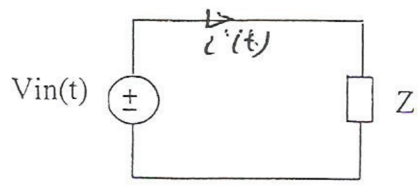
2- $Z(j\omega) = e^{j2\omega}$

$V_{in}(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt = \int_1^2 1 \, e^{-j\omega t} dt = \frac{1}{-j\omega} \left[e^{-j\omega t} \right]_1^2$

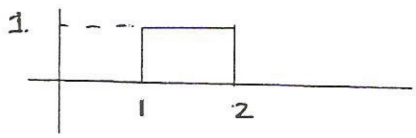
$V_{in}(j\omega) = \frac{1}{-j\omega} \left[e^{-j2\omega} - e^{-j\omega} \right]$

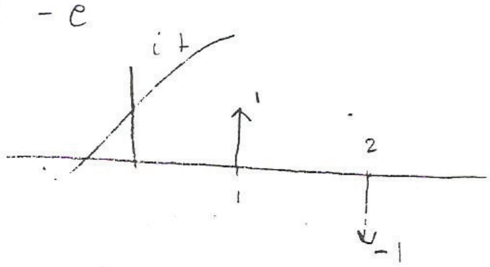
$I(j\omega) = \frac{V_{in}(j\omega)}{Z(j\omega)} = \frac{\frac{1}{-j\omega}\left[e^{-j2\omega} - e^{-j\omega} \right]}{\frac{1}{j\omega}} = e^{-j\omega} - e^{-j2\omega}$

$I(t) = \delta(t-1) - \delta(t-2)$



Vin(t)

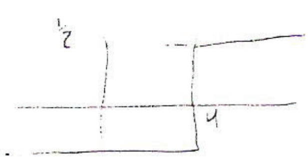


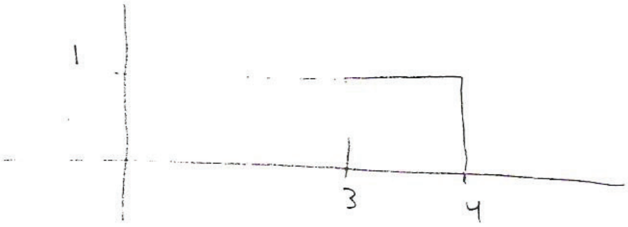


$I(j\omega) = \frac{1}{-j\omega} \left[e^{-j2\omega} - e^{-j\omega} \right] * e^{-j2\omega} = \frac{e^{-j4\omega}}{-j\omega+0} + \frac{e^{-j3\omega}}{j\omega+0}$

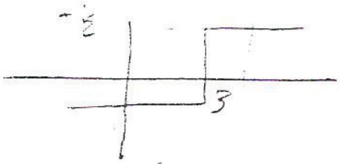
$I(t) = -\frac{1}{2} \operatorname{Sgn}(t-4) + \frac{1}{2} \operatorname{Sgn}(t-3)$

Sgn (t-4)





Sgn (t-3)



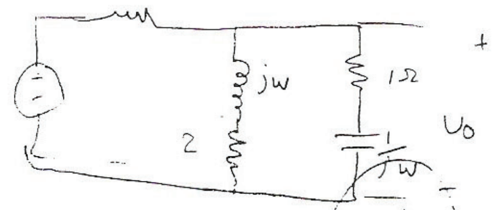
$3 < t < 4 \quad \frac{1}{2} - \frac{-1}{2} = 1$

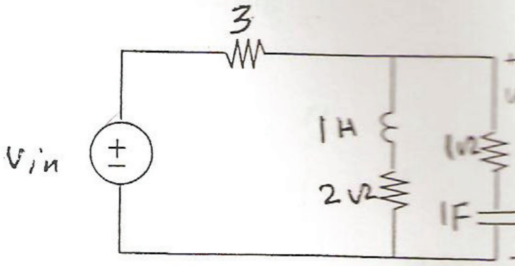
$\frac{1}{jwc}$

Question # (5 marks)

For system shown knowing that Vin(t) is the input and Vo(t) is the output:

Find h(t)





$$V_o = \frac{2+jw}{5+jw} \cdot V_{in} \Rightarrow \frac{V_o(jw)}{V_{in}(jw)} = H(jw) = \frac{2+jw}{5+jw}$$

$$H(jw) = 1 - \frac{3}{5+jw}$$

$$h(t) = (\delta(t) - 3e^{-5t})\, u(t)$$

$$\begin{array}{r} 1 \\ 5+jw \overline{)\, 2 + jw} \\ \mp 5 \mp jw \\ \hline -3 \end{array}$$