An Najah University – Faculty of engineering
Electrical Eng. Department
System and Signals

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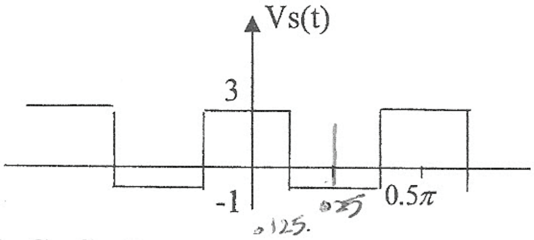
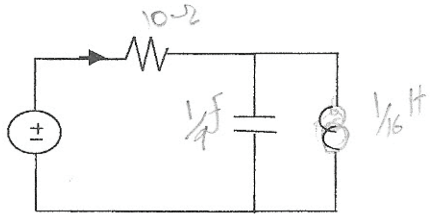
$\frac{10.5}{25}$

1st hr exam
6/10/2009

Question #1(9 marks)

The periodic signal shown applied to the following circuit
Where R = 10 ohm , L = 1/16 H C = 1/9F





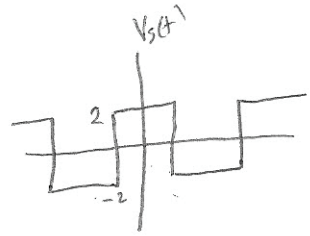
Find: 1- C_0, C_1, C_2, C_3

2- Power dissipated in the resistance due to first and third harmonics

n=1 n=3

$C_n = \frac{1}{T}\int f(t)\, e^{-jn\omega_0 t}\,dt$ $\quad C_n = C_0 + \sum C_n e^{jn\omega t}$

$1 +$



even: $C_n = \frac{2}{T}\int_0^{T/2} f(t)\cos n\omega_0 t\,dt$ $\quad$ where $T = 0.5\pi$

$\therefore \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{.5\pi} = 4$ $\quad$ even symmetry

$= \frac{2}{0.5\pi}\int_0^{\pi/4} f(t)\cos n4t\,dt$

$f(t) = \begin{cases} 2 & 0 < t < .125\pi \\ -2 & .125\pi < t < .375\pi \\ 2 & .375\pi < t < 0.5\pi \end{cases}$

$\therefore C_n = \frac{2}{0.5\pi}\left[\int_0^{.125\pi} 2\cos n4t\,dt + \int_{.125\pi}^{.25\pi} -2\cos n4t\,dt\right]$

$= \frac{2}{0.5\pi}\left[\frac{\cancel{2}\sin n4t}{n\cancel{4}2}\Big|_0^{.125\pi} + \frac{-\cancel{2}\sin n4t}{n\cancel{4}2}\Big|_{.125\pi}^{.25\pi}\right]$

$\frac{4}{9}$

$= \frac{2}{0.5\pi}\left[\left(\frac{\sin n.5\pi}{2n} - \cancel{\frac{\sin 0}{2n}}\right) + \left(\cancel{\frac{-\sin n\pi}{2n}} - \frac{-\sin n.5\pi}{2n}\right)\right]$

$= \frac{\cancel{2}}{0.5\pi}\cdot\frac{\sin n.5\pi - \sin n\pi + \sin n.5\pi}{\cancel{2}n} = \frac{2}{\pi}\cdot\frac{2\sin n.5\pi - \cancel{\sin n\pi}}{.5n}$

$\therefore C_n = \frac{4\sin n.5\pi}{\pi n}$

$\therefore C_0 = 1 + \cancel{\frac{4\sin 0}{\pi n}}$ $\quad \therefore C_0 = 1$

$C_1 = 1 + \frac{4\sin .5\pi}{\pi} = \boxed{1.0341}$ $\quad \boxed{C_1 =}$

$C_2 = 1 + \frac{4\sin\pi}{2\pi}$ $\quad \therefore \boxed{C_2 = 1}$

$C_3 = 1 + \frac{4\sin 3\cdot.5\pi}{3\pi}$ $\quad \boxed{C_3 = 1.344}$

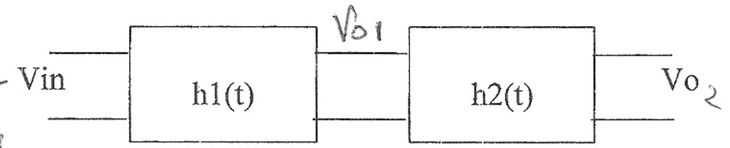
Question # 2(8 marks)

For linear system shown knowing that

$\boxed{Vo_2 = Vo}$

$h_1(t) = \delta(t-1) \Rightarrow h_1(jw) = e^{-jw}$

$h_2(t) = e^{-2t}u(t) \Rightarrow h_2(jw) = \dfrac{1}{jw+2}$



Find $V_o(t)$ if Vin(t) given by:

1- $Vin(t) = u(t) \Rightarrow Vin(jw) = \pi\delta(w) + \dfrac{1}{jw}$

2- $Vin(t) = 2\cos(2t+45^0) \xrightarrow{F} 2\pi[\delta(w-w_0)+\delta(w+w_0)]$

$V_{o1}(t) = Vin(t) \circledast h_1(t) = \int_{-\infty}^{+\infty} Vin(\lambda) * h_1(t-\lambda)\,d\lambda$

OR $= \int_{-\infty}^{\infty} h_1(\lambda) * Vin(t-\lambda)\,d\lambda$

$Vo_2(t) = Vo_1(t) \circledast h_2(t)$

(1) $Vo_1(jw) = Vin(jw)\,h_1(jw) = \left[\pi\delta(w) + \dfrac{1}{jw}\right]e^{-jw} = \pi\delta(w)e^{-jw} + \dfrac{e^{-jw}}{jw}$

$Vo_2(jw) = \left[\pi\delta(w)e^{-jw} + \dfrac{e^{-jw}}{jw}\right] * \dfrac{1}{jw+2} = \dfrac{\pi}{2}\delta(w)e^{-jw} + \dfrac{e^{-jw}}{jw(jw+2)}$

$\therefore \cancel{Vo(t)} = \dfrac{\pi}{2}\delta(w)e^{-jw} + \left[\dfrac{A}{jw} + \dfrac{B}{jw+2}\right]e^{-jw}$ $\quad A = \tfrac{1}{2},\ B = -\tfrac{1}{2}$

$Bjw + A(jw+2) = 1$

$jw = 0 \quad A = +\dfrac{1}{2}$

$jw = -2 \quad B = \dfrac{-1}{2}$

(1) $\therefore \boxed{Vo(t) = \tfrac{1}{2}u(t-1) - \tfrac{1}{2}e^{-2(t-1)}u(t-1)}$ ✓

(2) $Vo_1(jw) = 2[2\pi\delta(w-45) + 2\pi\delta(w+45)] * e^{-jw}$ $\quad \boxed{\text{Time shifting}}$

$Vo_1(t) = 2\cos(2(t-1)+45)$

$\dfrac{5.5}{8}$

$Vo_2(jw) = Vo_1(jw) * h_2(jw)$

$= [4\pi\delta(w-45) + 4\pi\delta(w+45)] * e^{-jw} * \dfrac{1}{jw+2}$

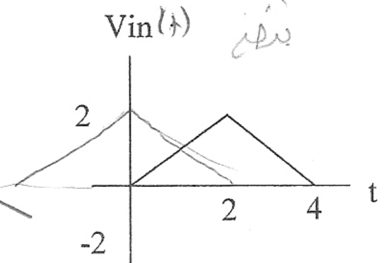
$= \left[\dfrac{4\pi\delta(w-45)}{jw+2} + \dfrac{4\pi\delta(w+45)}{jw+2}\right]e^{-jw}$

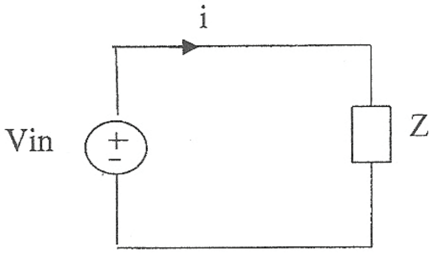
Question # 3(8 marks)

a) For system shown, knowing that Vin(t) given by the following signal , find and sketch i(t) if:

1- z(t) = sgn(t) $Z(jw) = jw$

2- z(jw) = e^{j2w}





$i(t) = \dfrac{Vin(t)}{Z(t)}$

$i(jw) = \dfrac{Vin(jw)}{Z(jw)}$ jw

I don't know Vin(jw) = ?

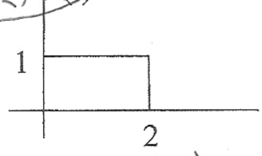
I can't transform this signal

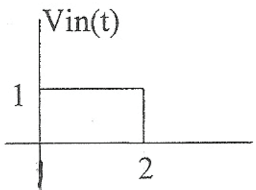
$\cancel{\boxed{Vin(t) = \begin{cases} t & 0<t<2 \\ 4-t & 2<t<4 \end{cases}}}$

b) Use Convolution to find Vo(t) = Vin(t) * h(t)

Knowing that h(t) and Vin(t) given by signals shown

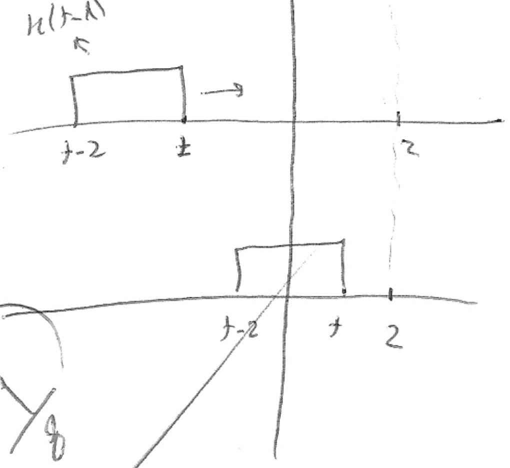
Sketch Vo(t)





(b)

$Vo(t) = \int_{-\infty}^{\infty} Vin(\lambda) * h(t-\lambda)\, d\lambda$



for $t<0$ => $Vo(t) = 0$

for $0<t<2$ => $Vo(t) = \int_0^2 1 * 1\, d\lambda$

$= \lambda \Big|_0^2 = 2$

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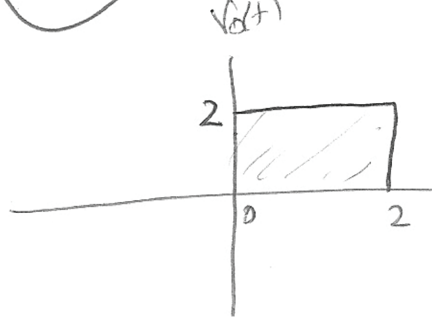
for $t>2$ => $Vo(t) = \int_2^{\infty} 0 \times 1\, d\lambda = 0$

$\cancel{\therefore Vo = \begin{cases} 0 & t<0 \\ 2 & 0<t<2 \\ 0 & t>0 \end{cases}}$ ⟹



$y - 0 = m(x-4)$

$m = \dfrac{0-2}{4-2} = \dfrac{-2}{2} = -1$

$y = -x+4$