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Question #1(12 marks)

a) 1- Find Z-inverse for $X(z) = \frac{1+2z^{-1}+z^{-2}}{1-3z^{-1}+2z^{-2}}$

$X(0) = \lim_{z \rightarrow \infty} X(z)$
 $X(\infty) = \lim_{z \rightarrow 1} (1-z^{-1}) X(z)$

- 2- Find Z- transform for $x(n) = u(n) - u(n-6)$
 3- Find $x(0)$ and $X(\infty)$ for function has the following

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 $\frac{z^{-1}+1}{z^{-1}-1}$
 $\frac{z^{-1}+1}{z^{-1}-1}$
 $\frac{z^{-1}+1}{z^{-1}-1}$

$X(z) = \frac{1+z^{-2}}{1-3z^{-1}+2z^{-2}}$

$X(z) = \frac{1+2z^{-1}+z^{-2}}{1-3z^{-1}+2z^{-2}} = \frac{(z^{-1}+1)(z^{-1}+1)}{(2z^{-1}-1)(z^{-1}-1)}$

$\frac{X(z)}{1} = \frac{A}{(2z^{-1}-1)} + \frac{B}{(z^{-1}-1)}$

$(z^{-1}+1)^2 = A(z^{-1}-1) + B(2z^{-1}-1)$

when $z^{-1} = 1$

$4 = B$

$B = 4$

when $z^{-1} = 2$

$2.25 = A(-0.5)$

$A = -4.5$

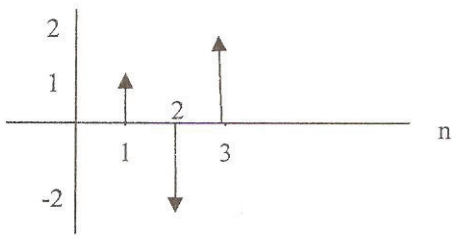
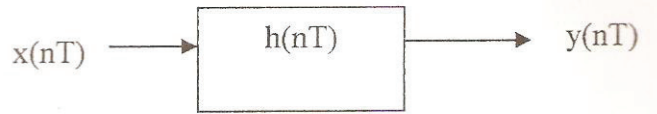
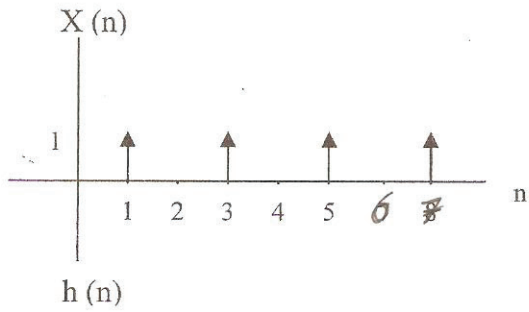
$X(z) = \frac{-4.5}{(2z^{-1}-1)} + \frac{4}{(z^{-1}-1)}$

$X(z) = \frac{4.5}{(1-2z^{-1})} - \frac{4}{(1-z^{-1})}$

$X(n) = 4.5(2)^n - 4$

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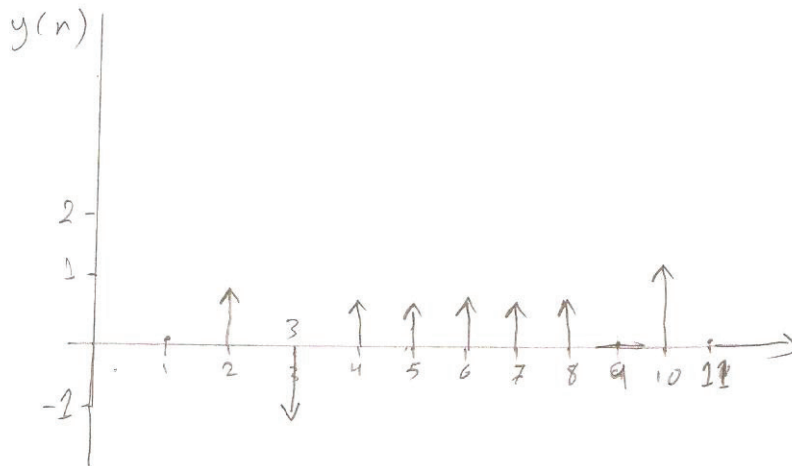
B- For system shown find $y(n)$, knowing that $x(n)$ and $h(n)$



$\frac{4}{6}$

					0	1	0	1	0	1	0	1
3	-2	1	0									
0	2	-2	1	0								
0	0	2	-2	1	0							
0	0	0	2	-2	1	0						
0	0	0	0	2	-2	1	0					
0	0	0	0	0	2	-2	1	0				
0	0	0	0	0	0	2	-2	1	0			
							2	-2	1	0		
								2	-2	1	0	
									2	-2	1	0
										2	-2	1
											2	-2

- $y(0) = 0$ ✓
- $y(1) = 0$ ✓
- $y(2) = 1$ ✓
- $y(3) = -1$ ✓
- $y(4) = 1$ ✓
- $y(5) = 1$ ✓
- $y(6) = 1$ ✓
- $y(7) = 1$ ✓
- $y(8) = 1$ ✓
- $y(9) = 0$
- $y(10) = 2$ ✓
- $y(11) = 0$ ✓



Question # 2 (8 marks)

- A certain filter has the following transfer function

$$H(j\omega) = \frac{0.01(j\omega + 10^3)^2}{(j\omega)^2 + 2 \times 10^2 j\omega + 10^4}$$

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- 1- Sketch Bode diagrams?
- 2- What is the kind and cutoff frequency (frequencies) for the filter?
- 3- For $V_{in}(t) = 2 \cos(500t + 30^\circ) + 2 \cos(1000t + 30^\circ)$ what is $V_o(t)$

$H(j\omega)$ can be written as

$$\frac{0.01 \times 10^2 \left(\frac{j\omega}{10^3} + 1\right)^2}{10^4 \left[\left(\frac{j\omega}{10^2}\right)^2 + 2\left(\frac{j\omega}{10^2}\right) + 1\right]} \Rightarrow \frac{1 \left(\frac{j\omega}{10^3} + 1\right)^2}{\left(\frac{j\omega}{10^2}\right)^2 + 2\left(\frac{j\omega}{10^2}\right) + 1} \Rightarrow \frac{\left(\frac{j\omega}{10^3}\right)^2 + 2\frac{j\omega}{10^3}}{\left(\frac{j\omega}{10^2}\right)^2 + 2\frac{j\omega}{10^2}}$$

* we can neglect the constant $K=1$ because it's magnitude = 0

for $\left[\left(\frac{j\omega}{10^3}\right)^2 + 2\left(\frac{j\omega}{10^3}\right) + 1\right]$

$\omega_n = 10^3$

$$|dB| = 20 \log \left(\frac{j\omega}{10^3}\right)^2 = 40 \log \left(\frac{j\omega}{10^3}\right) = \begin{cases} 40 \log \frac{\omega}{10^3} & \omega > 10^3 \\ 0 & \omega < 10^3 \end{cases}$$

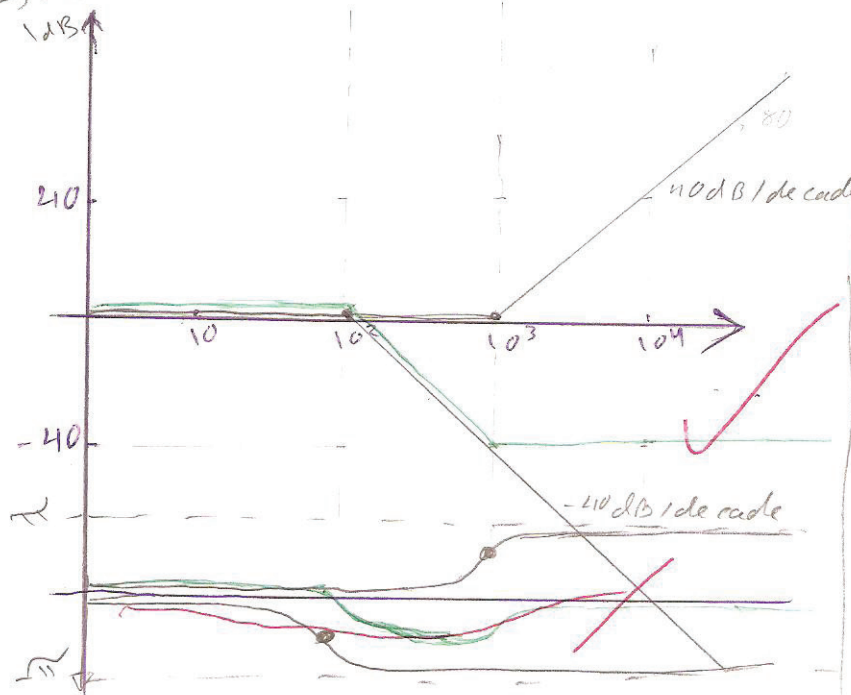
$\angle \Rightarrow$ will varies $0 \rightarrow \pi$

for $\left[\left(\frac{j\omega}{10^2}\right)^2 + 2\left(\frac{j\omega}{10^2}\right) + 1\right]$

$\omega_n = 10^2$

$$|dB| = -40 \log \frac{j\omega}{10^2} = \begin{cases} -40 \log \frac{\omega}{10^2} & \omega > 10^2 \\ 0 & \omega < 10^2 \end{cases}$$

$\angle \Rightarrow$ will varies $0 \rightarrow -\pi$



High pass filter
cutoff freq.
 $= 10^3$