

Chapter Fifteen:

Fourier Analysis Techniques

15.1 Find the exponential Fourier series for the periodic signal shown in Fig. P15.1.

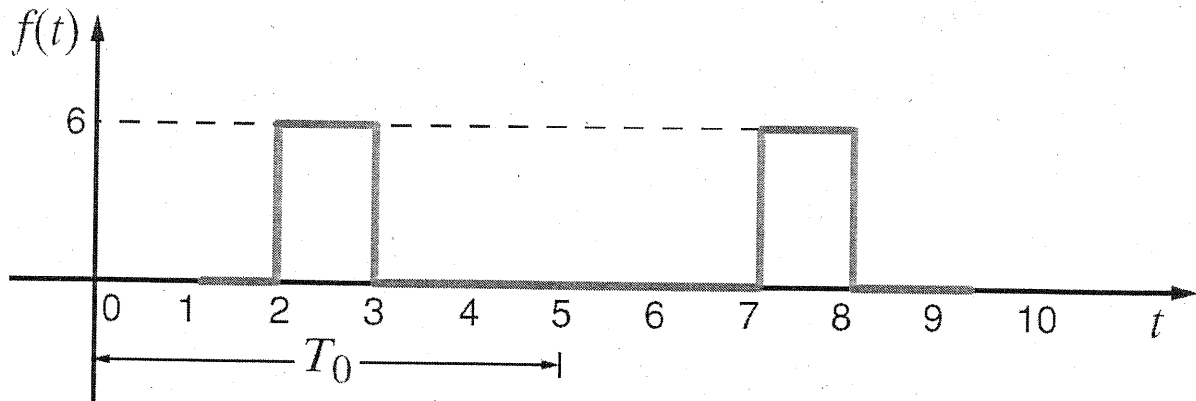


Figure P15.1

SOLUTION: $T_0 = 5 \text{ sec.}$ $\omega_0 T_0 = 2\pi$

$$C_n = \frac{1}{T_0} \int_0^{T_0/5} 6 e^{-jn\omega_0 t} dt = \frac{6}{jn2\pi} \left[e^{jn(\pi/5)} - e^{-jn(\pi/5)} \right]$$

$$C_n = \frac{6}{n\pi} e^{-jn(\pi/5)} \sin(n\pi/5) \quad \& \quad f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{6}{n\pi} e^{-jn\pi} \sin(n\pi/5) e^{j0.4\pi t}$$

15.2 Find the exponential Fourier series for the periodic pulse train shown in Fig. P15.2. **PSV**

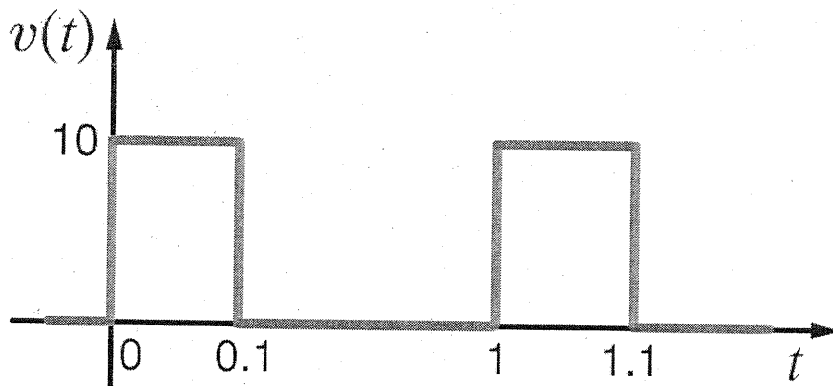


Figure P15.2

SOLUTION:

$$C_0 = \frac{10(0.1)}{1} = 1 \quad T_0 = 1 \quad \omega_0 = 2\pi$$

$$C_n = \frac{1}{T_0} \int_0^{T_0/10} 10 e^{-jn\omega_0 t} dt = \frac{10e^{-jn\pi/10}}{n\pi} \left[\frac{e^{jn\pi/10} - e^{-jn\pi/10}}{j2} \right]$$

$$C_n = \frac{10}{n\pi} e^{-jn\pi/10} \sin(n\pi/10)$$

$$f(t) = \frac{10}{\pi} \sum_{n=-\infty}^{\infty} \frac{1}{n} e^{-jn\pi/10} \sin(n\pi/10) e^{jn\omega_0 t}$$

15.3 Find the exponential Fourier series for the signal shown in Fig. P15.3. **CS**

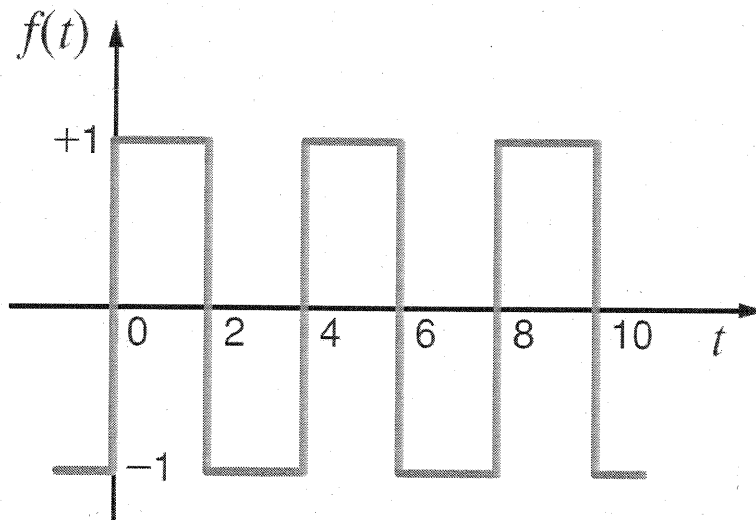


Figure P15.3

SOLUTION:

$$C_n = \frac{1}{T_0} \int_{-T_0/2}^0 -e^{-jn\omega_0 t} dt + \frac{1}{T_0} \int_0^{T_0/2} e^{-jn\omega_0 t} dt \quad \omega_0 = 2\pi/T_0$$

$$C_n = \frac{1}{T_0 jn\omega_0} \left[1 - e^{-jn\omega_0 T_0/2} + 1 - e^{jn\omega_0 T_0/2} \right] = \frac{1}{jn\pi} \left[z - e^{jn\pi} - e^{-jn\pi} \right]$$

$$C_n = \frac{1}{jn\pi} \left[z - 2 \cos(n\pi) \right] = \frac{1 - \cos(n\pi)}{jn\pi} = \begin{cases} 0 & n \text{ even} \\ \frac{z}{jn\pi} & n \text{ odd} \end{cases}$$

$$f(t) = \sum_{\substack{n=-\infty \\ n \text{ odd}}}^{\infty} \frac{z}{jn\pi} e^{jn\omega_0 t}$$

15.4 Find the exponential Fourier series for the signal shown in Fig. P15.4. CS

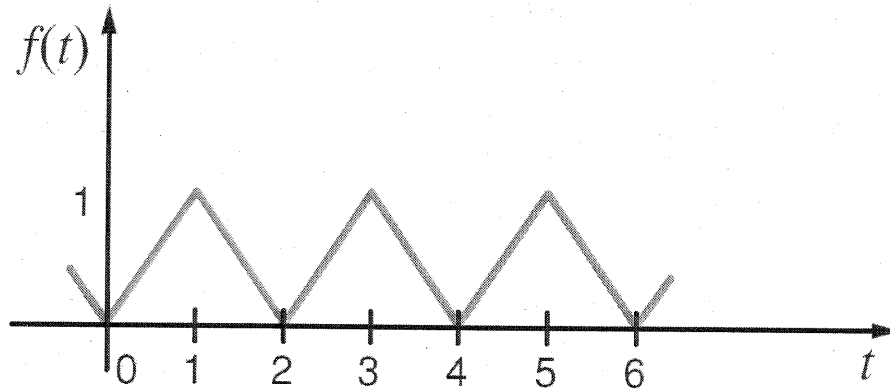


Figure P15.4

SOLUTION: $T_0 = 2 \text{ sec}$ $\omega_0 T_0 = 2\pi \Rightarrow \omega_0 = \pi$ let $a = jn\pi$

$$C_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} f(t) dt = \frac{1}{2} \int_{-1}^0 -t e^{-jn\omega_0 t} dt + \frac{1}{2} \int_0^1 t e^{-jn\omega_0 t} dt$$

$$C_n = \frac{1}{2} \left\{ \int_{-1}^0 -t e^{-at} dt + \int_0^1 t e^{-at} dt \right\} = \frac{1}{2} \left\{ \left(\frac{1}{a^2} + \frac{t}{a} \right) e^{-at} \Big|_{-1}^0 + \left(\frac{1}{a^2} + \frac{t}{a} \right) e^{-at} \Big|_0^1 \right\}$$

$$C_n = \frac{1}{2} \left\{ \frac{1}{a^2} - \left(\frac{1}{a^2} - \frac{1}{a} \right) e^a + \left(\frac{1}{a^2} \right) - \left(\frac{1}{a^2} + \frac{1}{a} \right) e^{-a} \right\}$$

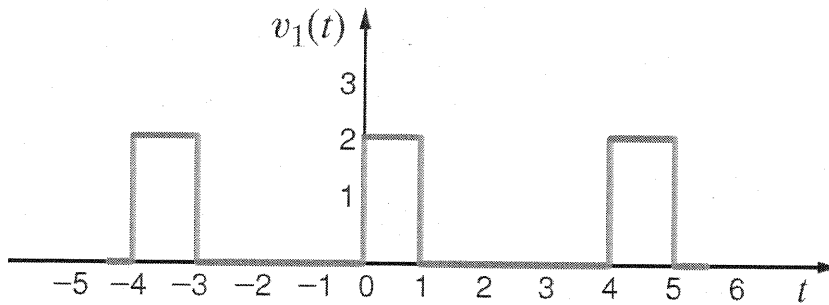
$$C_n = \frac{1}{2a^2} \left\{ 2 + e^a(a-1) - e^{-a}(a+1) \right\} = -\frac{1}{2n^2\pi^2} \left\{ 2 + e^{jn\pi}(jn\pi-1) - e^{-jn\pi}(jn\pi+1) \right\}$$

$$C_n = \frac{-2}{n^2\pi^2} \quad \text{if } n \text{ is odd} \quad C_n = 0 \quad \text{if } n \text{ is even}$$

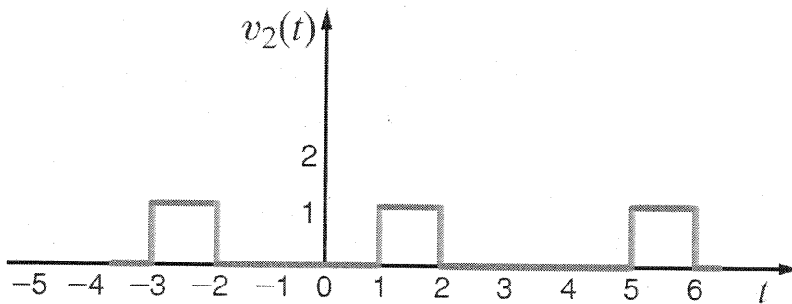
$$C_0 = 1/2$$

$$f(t) = \frac{1}{2} + \sum_{\substack{n=-\infty \\ n \neq 0 \\ n \text{ odd}}}^{\infty} \frac{-2}{n^2\pi^2} e^{jn\pi t}$$

15.5 Compute the exponential Fourier series for the waveform that is the sum of the two waveforms in Fig. P15.5 by computing the exponential Fourier series of the two waveforms and adding them.



(a)



(b)

Figure P15.5

SOLUTION: $T_0 = 4$ $\omega_0 T_0 = 2\pi$ $\Rightarrow \omega_0 = \pi/2$

$$C_{n1} = \frac{z}{T_0} \int_0^{T_0/4} e^{-jn\omega_0 t} dt = \frac{z}{jn2\pi} [1 - e^{-jn\pi/2}]$$

$$C_{n2} = \frac{C_{n1}}{2} e^{-jn\omega_0} \quad \text{Since } v_2(t) \text{ is half } v_1(t) \text{ shifted by 1 sec.}$$

$$C_n = C_{n1} + C_{n2} = \frac{1}{jn2\pi} \left\{ z - 2z e^{-jn\pi/2} + e^{-jn\pi/2} - e^{-jn\pi} \right\}$$

$$C_n = \frac{1}{jn\pi} \left\{ 1 - e^{-jn3\pi/4} \left[\frac{e^{jn\pi/4} + e^{-jn\pi/4}}{2} \right] \right\} = \frac{1}{jn\pi} (1 - e^{-jn3\pi/4} \cos(n\pi/4))$$

$$v(t) = \sum_{n=-\infty}^{\infty} \frac{1}{jn\pi} \left\{ 1 - e^{-jn3\pi/4} \cos(n\pi/4) \right\} e^{jn\pi t/2} \quad \checkmark \quad \checkmark$$

- 15.6 Given the waveform in Fig. P15.6, determine the type of symmetry that exists if the origin is selected at (a) l_1 and (b) l_2 .

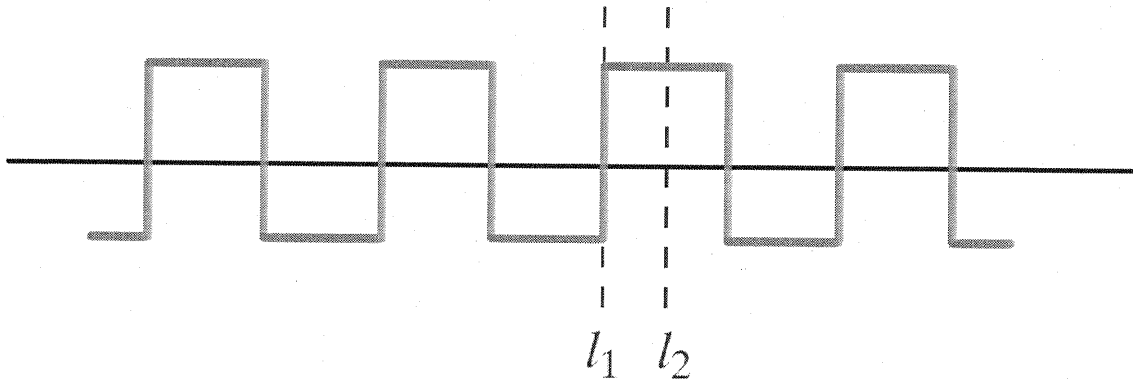
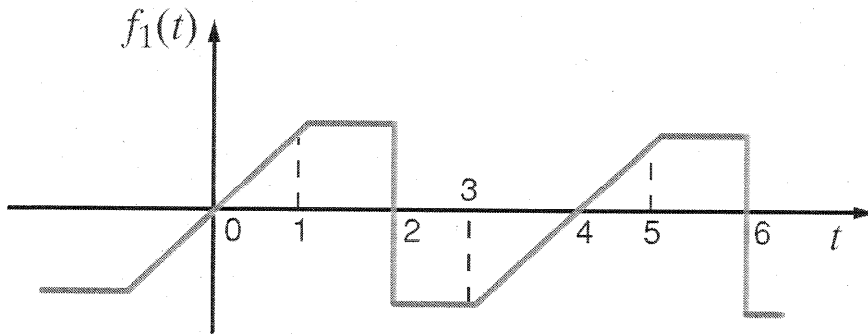


Figure P15.6

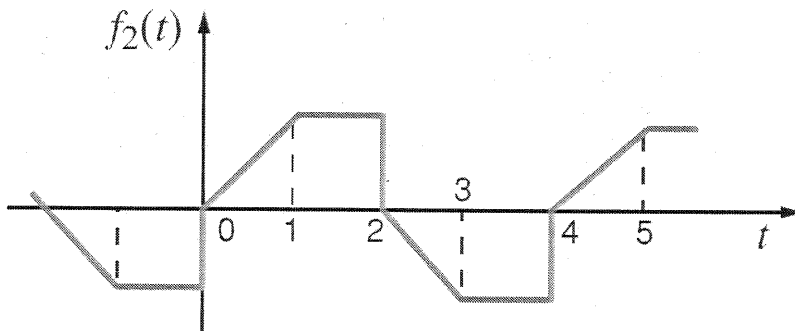
SOLUTION:

If origin is at l_1 , $v(t) = -v(-t)$ odd symmetry
 If origin is at l_2 , $v(t) = v(-t)$ even symmetry

15.7 What type of symmetry is exhibited by the two waveforms in Fig. P15.7?



(a)



(b)

Figure P15.7

SOLUTION:

(a) $f_1(t) = -f_1(-t)$ odd symmetry

(b) $f_2(t) = -f_2(t - T_0/2)$ half wave symmetry

15.8 Find the trigonometric Fourier series for the waveform shown in Fig. P15.8.

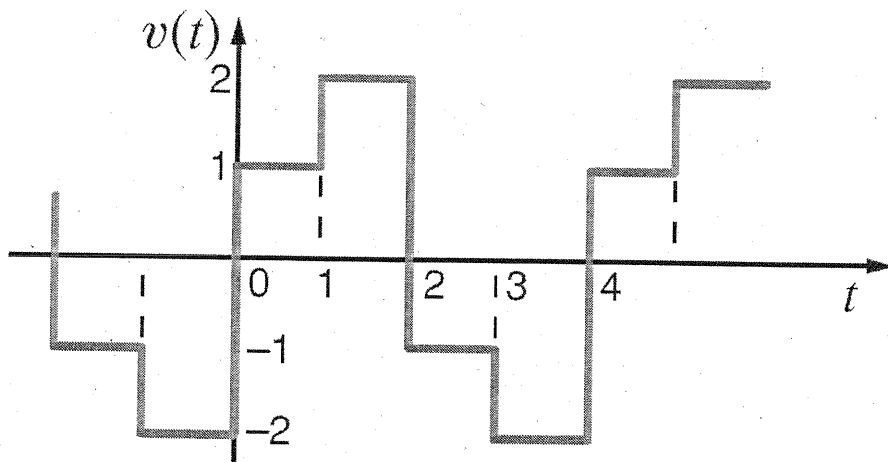


Figure P15.8

SOLUTION: $v(t)$ has half wave symmetry!

$$a_0 = 0 \quad T_0 = 4 \text{ sec} \quad \omega_0 = \pi/2 \quad a_n = b_n = 0 \text{ for } n \text{ even}$$

$$a_n = \frac{4}{T_0} \int_0^{T_0/2} f(t) \cos(n\omega_0 t) dt = \int_0^1 \cos(n\omega_0 t) dt + 2 \int_1^2 \cos(n\omega_0 t) dt$$

$$a_n = \frac{\sin(n\omega_0 t)}{n\omega_0} \Big|_0^1 + \frac{2\sin(n\omega_0 t)}{n\omega_0} \Big|_1^2 = \frac{\sin(n\pi/2) + 2\sin(n\pi) - 2\sin(n\pi/2)}{n\pi/2}$$

$$\text{for } n \text{ odd, } a_n = -\frac{2}{n\pi} \sin(n\pi/2)$$

$$b_n = \int_0^1 \sin(n\omega_0 t) dt + 2 \int_1^2 \sin(n\omega_0 t) dt = \frac{\cos(n\omega_0 t)}{n\omega_0} \Big|_0^1 + \frac{2\cos(n\omega_0 t)}{n\omega_0} \Big|_1^2$$

$$b_n = \frac{1 - \cos(n\pi/2) + 2\cos(n\pi/2) - 2\cos(n\pi)}{n\pi/2}$$

$$\text{for } n \text{ odd, } b_n = \frac{2}{n\pi} [1 - \cos(\pi/2) + 2\cos(\pi/2) - 2\cos(\pi)] = \frac{6}{n\pi}$$

$$v(t) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \left[\frac{3}{n} \sin(n\pi t/2) - \frac{\sin(n\pi/2)}{n} \cos(n\pi/2) \right] V \quad \checkmark$$

15.9 Find the trigonometric Fourier series for the periodic waveform shown in Fig. P15.9.

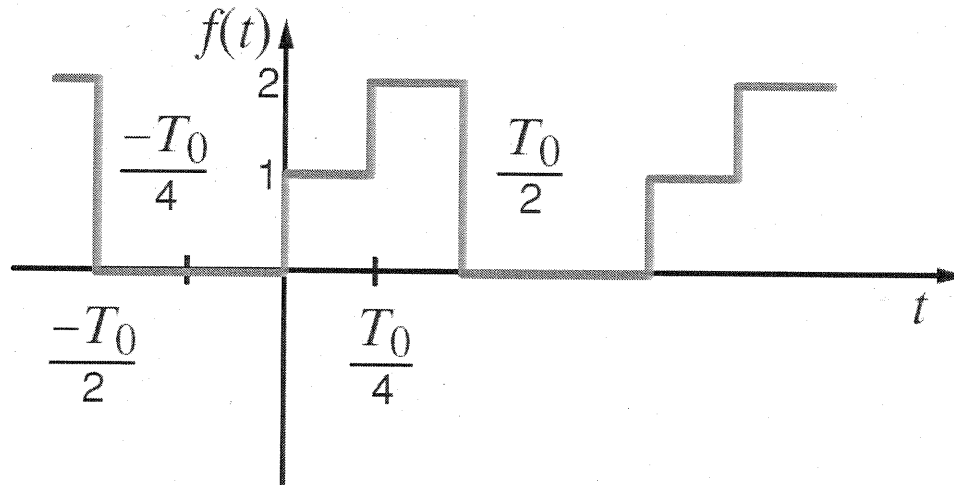


Figure P15.9

SOLUTION: $a_0 = \frac{1(T_0/4) + 2(T_0/4)}{T_0} = 3/4$

$$a_n = \frac{2}{T_0} \left\{ \int_0^{T_0/4} \cos(n\omega_0 t) dt + 2 \int_{T_0/4}^{T_0/2} \cos(n\omega_0 t) dt \right\}$$

$$a_n = \frac{2}{n\omega_0 T_0} \left\{ \sin(n\omega_0 t) \Big|_0^{T_0/4} + 2 \sin(n\omega_0 t) \Big|_{T_0/4}^{T_0/2} \right\} = -\frac{\sin(n\pi/2)}{n\pi}$$

$$b_n = \frac{2}{T_0} \left\{ \int_0^{T_0/4} \sin(n\omega_0 t) dt + 2 \int_{T_0/4}^{T_0/2} \sin(n\omega_0 t) dt \right\}$$

$$b_n = \frac{2}{n\omega_0 T_0} \left\{ \cos(n\omega_0 t) \Big|_{T_0/4}^0 + 2 \cos(n\omega_0 t) \Big|_{T_0/2}^{T_0/4} \right\} = \frac{1}{n\pi} \left[1 + \cos(n\pi/2) - 2\cos(n\pi) \right]$$

$$f(t) = \frac{3}{4} + \sum_{n=1}^{\infty} \frac{1}{n\pi} \left[(1 + \cos(n\pi/2) - 2\cos(n\pi)) \sin(n\omega_0 t) - \sin(n\pi/2) \cos(n\omega_0 t) \right] \checkmark$$

15.10 Given the waveform in Fig. P15.10 show that

$$f(t) = \frac{A}{2} + \sum_{n=1}^{\infty} \frac{-A}{n\pi} \sin \frac{2n\pi}{T_0} t \quad \text{PSV}$$

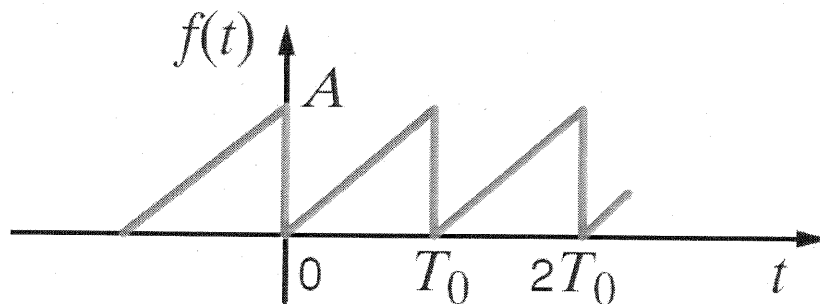


Figure P15.10

SOLUTION: $a_0 = A/2$ except for a_0 , odd symmetry, so $a_n = 0$ $n \neq 0$

$$b_n = \frac{2A}{T_0} \int_0^{T_0} \frac{t}{T_0} \sin(n\omega_0 t) dt = \frac{2A}{T_0^2} \left[\frac{\sin(n\omega_0 t)}{(n\omega_0)^2} - \frac{t \cos(n\omega_0 t)}{n\omega_0} \right]_0^{T_0}$$

$$b_n = \frac{2A}{T_0^2} \left[\frac{\sin(n2\pi)}{(n\omega_0)^2} - \frac{T_0 \cos(n2\pi)}{n\omega_0} \right] = \frac{-2A}{T_0} \left(\frac{1}{n\omega_0} \right) = \frac{-2A}{n2\pi} = \frac{-A}{n\pi}$$

$$f(t) = \frac{A}{2} + \sum_{n=1}^{\infty} \frac{-A}{n\pi} \sin \left(\frac{2n\pi}{T_0} t \right)$$

15.11 Find the trigonometric Fourier series coefficients for the waveform in Fig. P15.11. **CS**

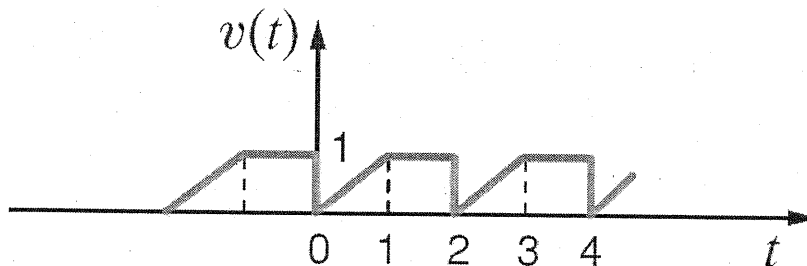


Figure P15.11

SOLUTION: $a_0 = \frac{(\frac{1}{2})(1)(1) + (1)(1)}{2} = \frac{3}{4}$ $T_0 = 2 \text{ sec}$ $\omega_0 = \pi$

$$a_n = \frac{2}{T_0} \int_0^1 t \cos(n\omega_0 t) dt + \frac{2}{T_0} \int_1^2 \cos(n\omega_0 t) dt = \int_0^1 t \cos(n\pi t) dt + \int_1^2 \cos(n\pi t) dt$$

$$a_n = \frac{\sin(n\pi)}{n\pi} + \frac{\cos(n\pi)}{(n\pi)^2} - \frac{1}{(n\pi)^2} + \frac{\sin(2n\pi)}{n\pi} - \frac{\sin(n\pi)}{n\pi} = \frac{\cos(n\pi) - 1}{(n\pi)^2}$$

$$b_n = \frac{2}{T_0} \left\{ \int_0^1 t \sin(n\omega_0 t) dt + \int_1^2 \sin(n\omega_0 t) dt \right\} = \int_0^1 t \sin(n\pi t) dt + \int_1^2 \sin(n\pi t) dt$$

$$b_n = \frac{\sin(n\pi)}{(n\pi)^2} - \frac{\cos(n\pi)}{n\pi} + \frac{\cos(n\pi)}{n\pi} - \frac{\cos(2n\pi)}{n\pi} = -\frac{1}{n\pi}$$

$a_0 = 3/4$	$a_n = \frac{\cos(n\pi) - 1}{(n\pi)^2}$	$b_n = -\frac{1}{n\pi}$
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15.12 Find the trigonometric Fourier series coefficients for the waveform in Fig. P15.12.

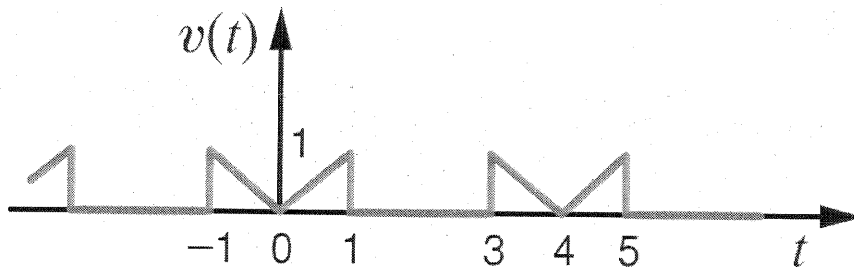


Figure P15.12

SOLUTION: $T_0 = 4$ $\omega_0 = \pi/2$ $a_0 = \frac{1}{4} \left[\frac{1}{2}(1)(1) + \frac{1}{2}(1)(1) \right] = 1/4$

Even symmetry, $b_n = 0$

$$a_n = \frac{4}{T_0} \int_0^{T_0/2} v(t) \cos(n\omega_0 t) dt = \int_0^1 t \cos(n\omega_0 t) dt = \left[\frac{\cos(n\omega_0 t)}{(n\omega_0)^2} + \frac{t \sin(n\omega_0 t)}{n\omega_0} \right]_0^1$$

$$a_n = \frac{\cos(n\pi/2) - 1}{(n\pi/2)^2} + \frac{\sin(n\pi/2)}{n\pi/2} = \frac{4}{(n\pi)^2} (\cos(n\pi/2) - 1) + \frac{2}{n\pi} \sin(n\pi/2)$$

$$a_0 = 1/4 \quad b_n = 0 \quad a_n = \frac{4}{(n\pi)^2} (\cos(n\pi/2) - 1) + \frac{2}{n\pi} \sin(n\pi/2)$$

15.13 Find the trigonometric Fourier series coefficients for the waveform in Fig. P15.13.

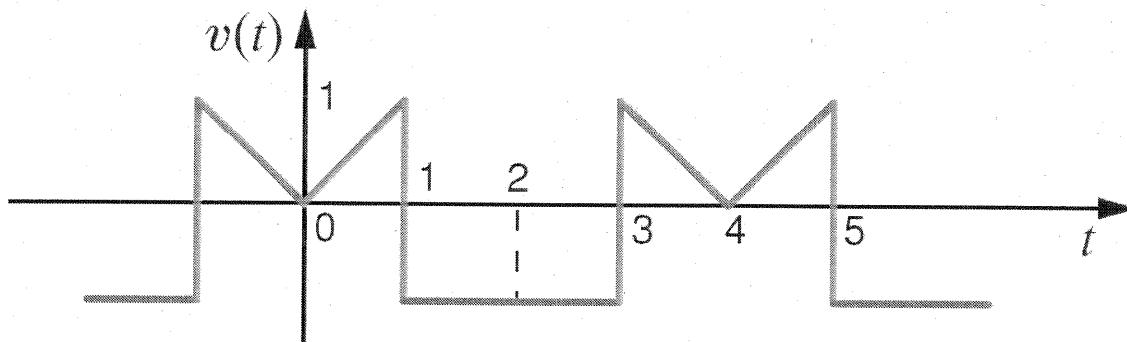


Figure P15.13

SOLUTION: $T_0 = 4$ $\omega_0 = \pi/2$ $a_0 = \frac{\frac{1}{2}(1)(1) - 2(1) + \frac{1}{2}(1)(1)}{4} = -\frac{1}{4}$

even symmetry, so $b_n = 0$

$$a_n = \frac{4}{T_0} \int_0^{T_0/2} v(t) \cos(n\omega_0 t) dt = \int_0^1 t \cos(n\omega_0 t) dt - \int_1^2 \cos(n\omega_0 t) dt$$

$$a_n = \left[\frac{\cos(n\omega_0 t)}{(n\omega_0)^2} + \frac{t \sin(n\omega_0 t)}{n\omega_0} \right]_0^1 - \frac{\sin(n\omega_0 t)}{n\omega_0} \Big|_1^2$$

$$a_n = \frac{\cos(n\omega_0) - 1}{(n\omega_0)^2} + \frac{\sin(n\omega_0)}{n\omega_0} - \frac{\sin(2n\omega_0)}{n\omega_0} + \frac{\sin(n\omega_0)}{n\omega_0}$$

$$a_n = \frac{4}{(n\pi)^2} (\cos(n\pi/2) - 1) + \frac{4}{n\pi} \sin(n\pi/2) \quad a_0 = -\frac{1}{4} \quad b_n = 0$$

15.14 Find the trigonometric Fourier series coefficients for the waveform in Fig. P15.14.

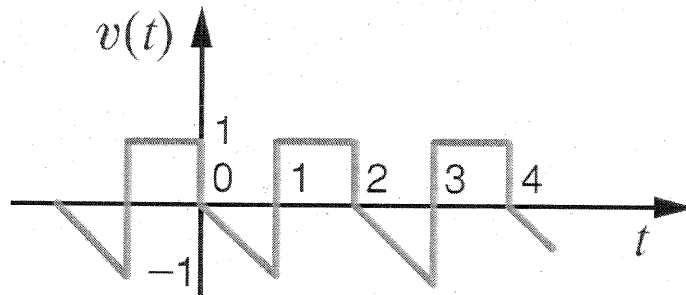


Figure P15.14

SOLUTION: $T_0 = 2 \text{ sec}$ $\omega_0 = \pi \text{ r/s}$ $a_0 = -\frac{(1/2)(1)(1) + (1)(1)}{2} = \frac{1}{4}$

$$a_n = \frac{2}{T_0} \left\{ \int_0^1 -t \cos(n\omega_0 t) dt + \int_1^2 \cos(n\omega_0 t) dt \right\} = \frac{\cos(n\omega_0 t)}{(n\pi)^2} + \frac{t \sin(n\omega_0 t)}{n\omega_0} \Big|_0^1 + \frac{\sin(n\omega_0 t)}{n\omega_0} \Big|_1^2$$

$$a_n = \frac{1 - \cos(n\pi)}{(n\pi)^2} - \frac{\sin(n\pi)}{n\pi} + \frac{\sin(2n\pi)}{n\pi} - \frac{\sin(n\pi)}{n\pi} = \frac{1 - \cos(n\pi)}{(n\pi)^2}$$

$$b_n = \frac{2}{T_0} \left\{ \int_0^1 -t \sin(n\omega_0 t) dt + \int_1^2 \sin(n\omega_0 t) dt \right\} = \frac{t \cos(n\pi t)}{n\pi} - \frac{\sin(n\pi t)}{(n\pi)^2} \Big|_0^1 + \frac{\cos(n\pi t)}{n\pi} \Big|_1^2$$

$$b_n = \frac{\cos(n\pi)}{n\pi} - \frac{\sin(n\pi)}{(n\pi)^2} + \frac{\cos(2n\pi)}{n\pi} - \frac{\cos(n\pi)}{n\pi} = \frac{2\cos(n\pi) - 1}{n\pi}$$

$a_0 = 1/4$	$a_n = \frac{1 - \cos(n\pi)}{(n\pi)^2}$	$b_n = \frac{2\cos(n\pi) - 1}{n\pi}$
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15.15 Find the trigonometric Fourier series coefficients for the waveform in Fig. P15.15.

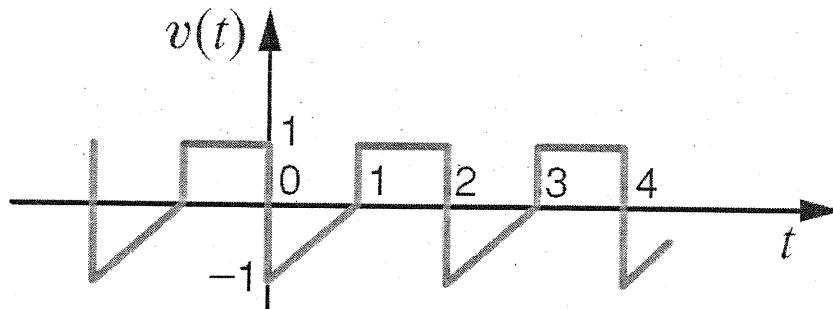


Figure P15.15

SOLUTION: $T_0 = 2 \text{ sec}$ $\omega_0 = \pi \text{ r/s}$ $a_0 = \frac{1}{2} \left[-\frac{1}{2}(1)(1) + (1)(1) \right] = 1/4$

$$a_n = \frac{2}{T_0} \left\{ \int_0^1 (t-1) \cos(n\pi t) dt + \int_1^2 \cos(n\pi t) dt \right\}$$

$$a_n = \left(\frac{\cos(n\pi t)}{(n\pi)^2} + t \frac{\sin(n\pi t)}{n\pi} \right) \Big|_0^1 - \frac{\sin(n\pi t)}{n\pi} \Big|_0^1 + \frac{\sin(n\pi t)}{n\pi} \Big|_1^2$$

$$a_n = \frac{\cos(n\pi) - 1}{(n\pi)^2} + \frac{\sin(n\pi)}{n\pi} - \frac{\sin(n\pi)}{n\pi} + \frac{\sin(2n\pi)}{n\pi} - \frac{\sin(n\pi)}{n\pi} = \frac{\cos(n\pi) - 1}{(n\pi)^2}$$

$$b_n = \int_0^1 (t-1) \sin(n\pi t) dt + \int_1^2 \sin(n\pi t) dt$$

$$b_n = \left(\frac{\sin(n\omega_0 t)}{(n\omega_0)^2} - \frac{t \cos(n\omega_0 t)}{n\omega_0} \right) \Big|_0^1 + \frac{\cos(n\omega_0 t)}{n\omega_0} \Big|_2^1 + \frac{\cos(n\omega_0 t)}{n\omega_0} \Big|_1^2$$

$$b_n = \frac{\sin(n\pi)}{(n\pi)^2} - \frac{\cos(n\pi)}{n\pi} + \frac{\cos(n\pi)}{n\pi} - \frac{\cos(2n\pi)}{n\pi} + \frac{\cos(n\pi)}{n\pi} - \frac{1}{n\pi} = \frac{\cos(n\pi) - 2}{n\pi}$$

$a_0 = 1/4$	$a_n = \frac{\cos(n\pi) - 1}{(n\pi)^2}$	$b_n = \frac{\cos(n\pi) - 2}{n\pi}$
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15.16 Derive the trigonometric Fourier series for the waveform shown in Fig. P15.16.

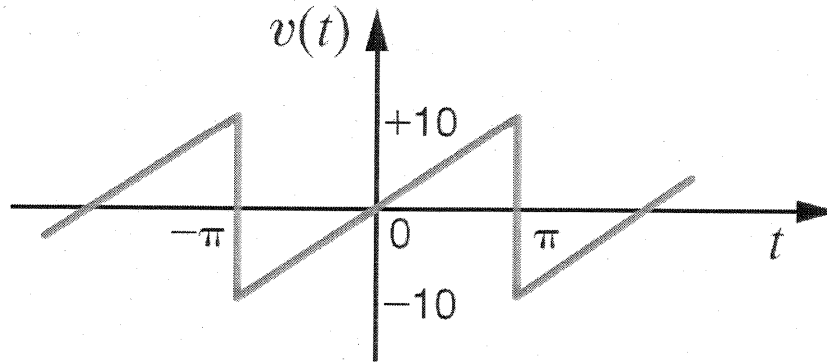


Figure P15.16

SOLUTION: $a_0 = 0$ $T_0 = 2\pi$ $\omega_0 = 1 \text{ rad/s}$

odd symmetry $\Rightarrow a_n = 0$

$$b_n = \frac{4}{T_0} \int_0^{T_0/2} \frac{10}{\pi} t \sin(nt) dt = \frac{40}{\pi^2} \int_0^{\pi/2} t \sin(nt) dt$$

$$b_n = \frac{40}{\pi^2} \left[\frac{\sin(nt)}{n^2} - \frac{t \cos(nt)}{n} \right]_0^{\pi} = \frac{40}{\pi^2} \left[\frac{\sin(n\pi)}{n^2} - \frac{\pi \cos(n\pi)}{2n} \right]$$

$$b_n = \frac{40}{\pi^2} \left[-\frac{\pi \cos(n\pi)}{2n} \right] = -\frac{20}{n\pi} \cos(n\pi) = (-1)^{n+1} \left(\frac{20}{n\pi} \right)$$

$$v(t) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{20}{n\pi} \sin(nt)$$

15.17 Find the trigonometric Fourier series coefficients for the waveform in Fig. P15.17.

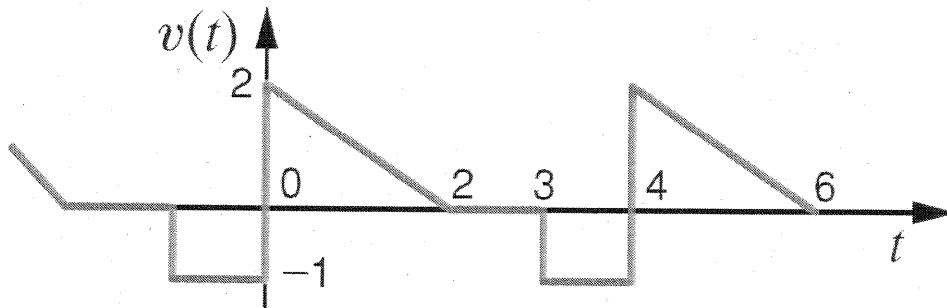


Figure P15.17

SOLUTION: $T_0 = 4$ $\omega_0 = \pi/2$ $a_0 = \frac{1}{4} \left[\frac{1}{2} (2)(2) - (1)(1) \right] = 1/4$

$$a_n = \frac{2}{T_0} \left\{ \int_0^2 (2-t) \cos(n\omega_0 t) dt - \int_3^4 \cos(n\omega_0 t) dt \right\}$$

$$a_n = \frac{\sin(n\omega_0 t)}{n\omega_0} \Big|_0^2 + \left(\frac{t \sin(n\omega_0 t)}{2n\omega_0} + \frac{\cos(n\omega_0 t)}{2(n\omega_0)^2} \right) \Big|_2^0 + \frac{\sin(n\omega_0 t)}{2n\omega_0} \Big|_3^4$$

$$a_n = \frac{\sin(n\pi)}{n\omega_0} + \frac{1}{2(n\omega_0)^2} - \frac{\sin(n\pi)}{n\omega_0} - \frac{\cos(n\pi)}{2(n\omega_0)^2} + \frac{\sin(n3\pi/2)}{2n\omega_0} - \frac{\sin(n2\pi)}{2n\omega_0}$$

$$a_n = \frac{2}{(n\pi)^2} (1 - \cos(n\pi)) - \frac{\sin(n\pi/2)}{n\pi}$$

$$a_0 = 1/4$$

$$b_n = \frac{2}{T_0} \left\{ \int_0^2 (2-t) \sin(n\omega_0 t) dt + \int_3^4 -\sin(n\omega_0 t) dt \right\}$$

$$b_n = \frac{\cos(n\omega_0 t)}{n\omega_0} \Big|_2^0 + \left(\frac{t \cos(n\omega_0 t)}{2n\omega_0} - \frac{\sin(n\omega_0 t)}{2(n\omega_0)^2} \right) \Big|_0^2 + \frac{\cos(n\omega_0 t)}{2n\omega_0} \Big|_3^4$$

$$b_n = \frac{1 - \cos(n\pi)}{n\omega_0} + \frac{\cos(n\pi)}{n\omega_0} - \frac{\sin(n\pi)}{2(n\omega_0)^2} + \frac{\cos(n2\pi) - \cos(n3\pi/2)}{2n\omega_0}$$

$$b_n = \frac{3 - \cos(n\pi/2)}{n\pi}$$

15.18 Find the trigonometric Fourier series for the waveform shown in Fig. P15.18. **CS**

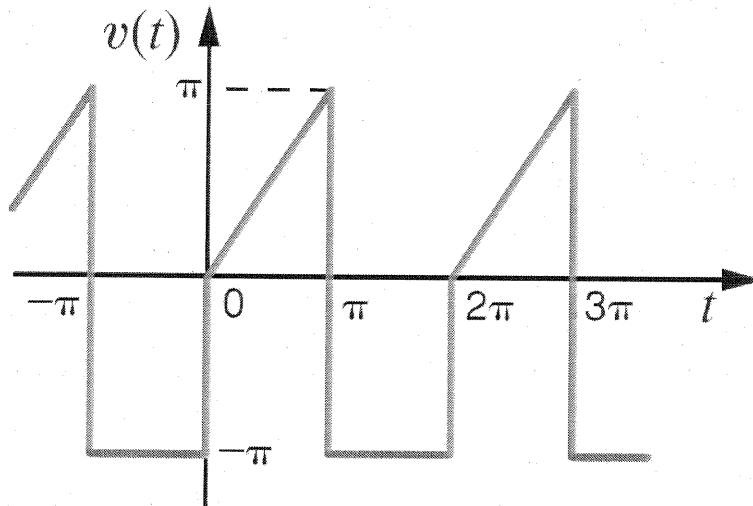


Figure P15.18

SOLUTION: $T_0 = 2\pi$ $\omega_0 = 1$ r/s $a_0 = \frac{1}{2\pi} [\pi^2/2 - \pi^2] = -\pi/4$

$$a_n = \frac{1}{\pi} \int_0^{\pi} t \cos(n\omega_0 t) dt - \frac{1}{\pi} \int_{\pi}^{2\pi} \pi \cos(n\omega_0 t) dt$$

$$a_n = \frac{1}{\pi} \left[\frac{\cos(n\omega_0 t)}{(n\omega_0)^2} + \frac{t \sin(n\omega_0 t)}{n\omega_0} \right] \Big|_0^{\pi} - \left[\frac{\sin(n\omega_0 t)}{n\omega_0} \right] \Big|_{\pi}^{2\pi}$$

$$a_n = \frac{1}{\pi} \left[\frac{\cos(n\pi)}{n^2} - \frac{1}{n^2} + \frac{\pi \sin(n\pi)}{n} \right] - \frac{\sin(n 2\pi)}{n} + \frac{\sin(n\pi)}{n} = \frac{1}{\pi n^2} [\cos(n\pi) - 1]$$

$$b_n = \frac{1}{\pi} \int_0^{\pi} t \sin(n\omega_0 t) dt - \int_{\pi}^{2\pi} \pi \sin(n\omega_0 t) dt$$

$$b_n = \frac{1}{\pi} \left[\frac{\sin(n\omega_0 t)}{(n\omega_0)^2} - \frac{t \cos(n\omega_0 t)}{n\omega_0} \right] \Big|_0^{\pi} + \frac{\cos(n\omega_0 t)}{n\omega_0} \Big|_{\pi}^{2\pi}$$

$$b_n = \frac{1}{\pi} \left[\frac{\sin(n\pi)}{n^2 \pi^2} - \frac{\pi \cos(n\pi)}{n\omega_0} \right] + \frac{\cos(2n\pi) - \cos(n\pi)}{n\omega_0} = \frac{1}{n} (1 - 2 \cos(n\pi))$$

$$v(t) = -\frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{1}{\pi n^2} (\cos(n\pi) - 1) \cos(nt) + \frac{1}{n} (1 - 2 \cos(n\pi)) \sin(nt) \quad \checkmark \checkmark$$

15.19 Derive the trigonometric Fourier series for the function shown in Fig. P15.19. **CS**

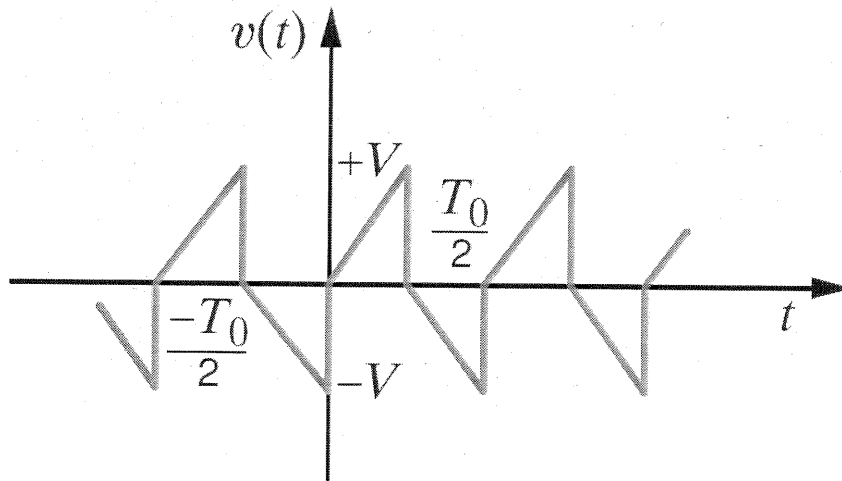


Figure P15.19

SOLUTION: Half wave symmetry $\Rightarrow a_n = b_n = 0$ for n even & $a_0 = 0$

$$a_n = \frac{4}{T_0} \int_0^{T_0/2} v(t) \cos(n\omega_0 t) dt = \frac{4}{T_0} \int_0^{T_0/2} \frac{2V}{T_0} t \cos(n\omega_0 t) dt = \frac{8V}{T_0^2} \int_0^{T_0/2} t \cos(n\omega_0 t) dt$$

$$a_n = \frac{8V}{T_0^2} \left[\frac{t \sin(n\omega_0 t)}{n\omega_0} + \frac{\cos(n\omega_0 t)}{(n\omega_0)^2} \right] \Big|_0^{T_0/2} = \frac{8V}{T_0^2} \left[\frac{T_0}{2} \left(\frac{\sin(n\pi)}{n\omega_0} \right) + \frac{\cos(n\pi) - 1}{(n\omega_0)^2} \right]$$

$$a_n = 8V \left(\frac{\cos(n\pi) - 1}{4(n\pi)^2} \right) \text{ for } n \text{ odd, } a_n = -\frac{4V}{n^2\pi^2}$$

$$b_n = \frac{4}{T_0} \int_0^{T_0/2} v(t) \sin(n\omega_0 t) dt = \frac{8V}{T_0^2} \int_0^{T_0/2} t \sin(n\omega_0 t) dt$$

$$b_n = \frac{8V}{T_0^2} \left[\frac{\sin(n\omega_0 t)}{n\omega_0} - \frac{t \cos(n\omega_0 t)}{n\omega_0} \right] \Big|_0^{T_0/2} = \frac{8V}{T_0^2} \left\{ \frac{\sin(n\pi)}{n\omega_0} - \frac{T_0}{2} \frac{\cos(n\pi)}{n\omega_0} \right\}$$

$$b_n = -\frac{2V}{n\pi} \cos(n\pi) \text{ for } n \text{ odd, } b_n = \frac{2V}{n\pi}$$

$$v(t) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \frac{2V}{n\pi} \sin(n\omega_0 t) - \frac{4V}{n^2\pi^2} \cos(n\omega_0 t) \quad V$$

15.20 Derive the trigonometric Fourier series for the function $v(t) = A|\sin t|$ as shown in Fig. P15.20.

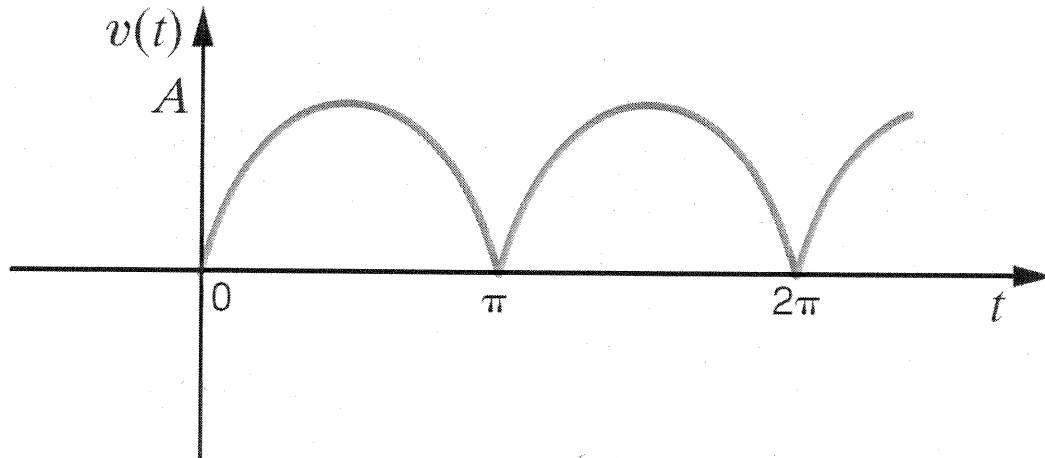


Figure P15.20

SOLUTION: $T_0 = \pi$ $\omega_0 = 2$ even function $\Rightarrow b_n = 0$

$$a_0 = \frac{1}{T_0} \int_0^{T_0} A \sin(t) dt = \frac{A}{T_0} \left[\cos(t) \right] \Big|_0^{T_0} = \frac{2A}{T_0} = \frac{2A}{\pi}$$

$$a_n = \frac{4}{T_0} \int_0^{T_0/2} A \sin(t) \cos(n\omega_0 t) dt = \frac{4A}{\pi} \int_0^{\pi/2} \sin(t) \cos(2nt) dt$$

$$\sin(t) \cos(2nt) = \left(\sin[(2n+1)t] + \sin(1-2n)t \right) / 2$$

$$a_n = \frac{2A}{\pi} \left[\frac{\cos(1+2n)t}{1+2n} + \frac{\cos(1-2n)t}{1-2n} \right] \Big|_0^{\pi/2}$$

$$a_n = \frac{2A}{\pi} \left[\frac{1 - \cos[(1+2n)\pi/2]}{1+2n} + \frac{1 - \cos[(1-2n)\pi/2]}{1-2n} \right] = \frac{2A}{\pi} \left[\frac{1}{1+2n} + \frac{1}{1-2n} \right]$$

$$a_n = \frac{4A}{\pi(1-4n^2)}$$

$$v(t) = \frac{2A}{\pi} \left[1 + \sum_{n=1}^{\infty} \left(\frac{2}{1-4n^2} \right) \cos(n2t) \right]$$

15.21 Derive the trigonometric Fourier series for the waveform shown in Fig. P15.21.

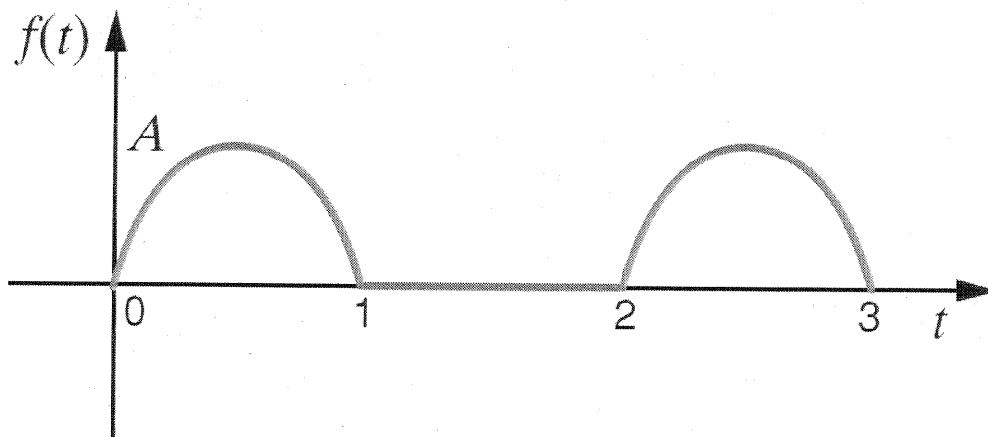


Figure P15.21

SOLUTION: $T_0 = 2$ $\omega_0 = \pi$

$$a_0 = \frac{1}{2} \int_0^1 A \sin(\pi t) dt = \frac{A}{2\pi} \cos(\pi t) \Big|_0^1 = A/\pi$$

$$a_n = A \int_0^1 \sin(\pi t) \cos(n\pi t) dt = \frac{A}{2} \int_0^1 [\sin(1-n)\pi t + \sin(1+n)\pi t] dt$$

$$a_n = \frac{A}{2} \left[\frac{\cos(1-n)\pi t}{(1-n)\pi} + \frac{\cos(1+n)\pi t}{(1+n)\pi} \right] \Big|_0^1 = \frac{A}{2} \left[\frac{1 - \cos(1-n)\pi}{(1-n)\pi} + \frac{1 - \cos(1+n)\pi}{(1+n)\pi} \right]$$

for n odd, $a_n = 0$ $a_n = \frac{2A}{\pi(1-n^2)}$ for n even

$$b_n = A \int_0^1 \sin(\pi t) \sin(n\pi t) dt = \begin{cases} 0 & n \neq 1 \\ A \int_0^1 \sin^2(\pi t) dt & \text{for } n=1 \end{cases}$$

$$b_1 = \frac{A}{2} \int_0^1 (1 - \cos(2\pi t)) dt = \frac{A}{2} \left[t - \frac{\sin(2\pi t)}{2\pi} \right] \Big|_0^1 = \frac{A}{2} [1] = A/2$$

$$f(t) = \frac{A}{\pi} + \frac{A}{2} \sin(\pi t) + \sum_{\substack{n=2 \\ n \text{ even}}}^{\infty} \frac{2A}{\pi(1-n^2)} \cos(n\pi t)$$

15.22 Use PSPICE to determine the Fourier series of the waveform in Fig. P15.22 in the form

$$v_s(t) = a_0 + \sum_{n=1}^{\infty} b_n \sin(n\omega_t + \theta_n)$$

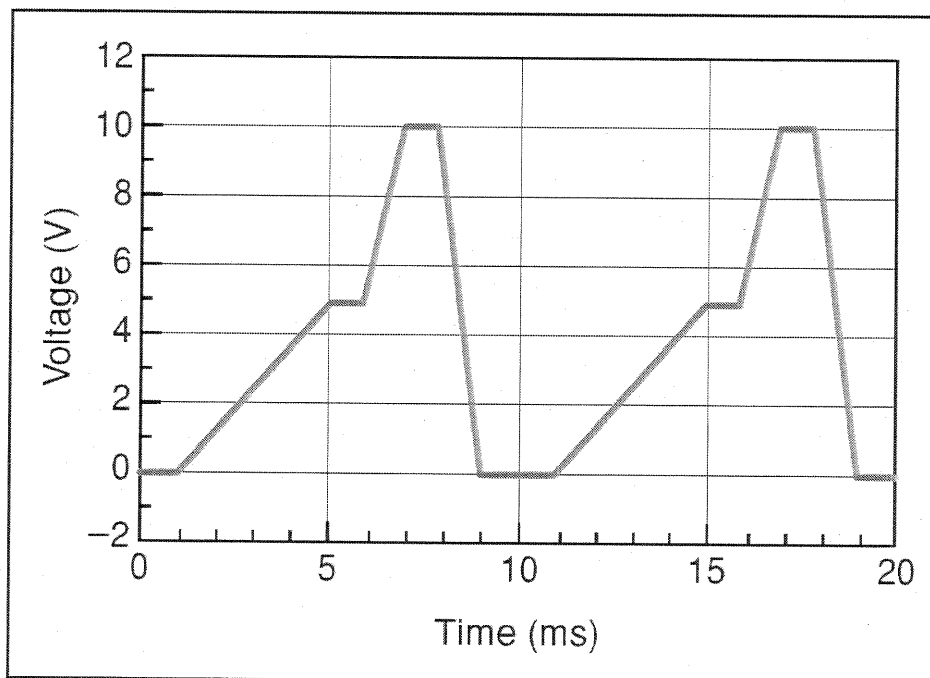


Figure P15.22

SOLUTION: Final time = 10 ms Step ceiling = 10 μ s Center freq = 100 kHz
 Number of harmonics = 10.

Use VPWL source to produce $v_s(t)$

15.22

FOURIER COMPONENTS OF TRANSIENT RESPONSE V(Vs)

DC COMPONENT = 3.750000E+00

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	1.000E+02	3.954E+00	1.000E+00	-1.424E+02	0.000E+00
2	2.000E+02	2.016E+00	5.099E-01	-8.774E+01	1.971E+02
3	3.000E+02	1.247E+00	3.154E-01	-8.735E+00	4.185E+02
4	4.000E+02	6.417E-01	1.623E-01	6.991E+01	6.395E+02
5	5.000E+02	2.027E-01	5.126E-02	9.000E+01	8.020E+02
6	6.000E+02	2.852E-01	7.213E-02	1.101E+02	9.645E+02
7	7.000E+02	2.291E-01	5.793E-02	-1.713E+02	8.256E+02
8	8.000E+02	1.260E-01	3.188E-02	-9.226E+01	1.047E+03
9	9.000E+02	4.883E-02	1.235E-02	-3.759E+01	1.244E+03
10	1.000E+03	1.993E-08	5.040E-09	-1.206E+02	1.303E+03

TOTAL HARMONIC DISTORTION = 6.310255E+01 PERCENT

15.23 Use PSPICE to determine the Fourier series of the waveform in Fig. P15.23 in the form

$$i_S(t) = a_0 + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t + \theta_n) \quad \text{CS}$$

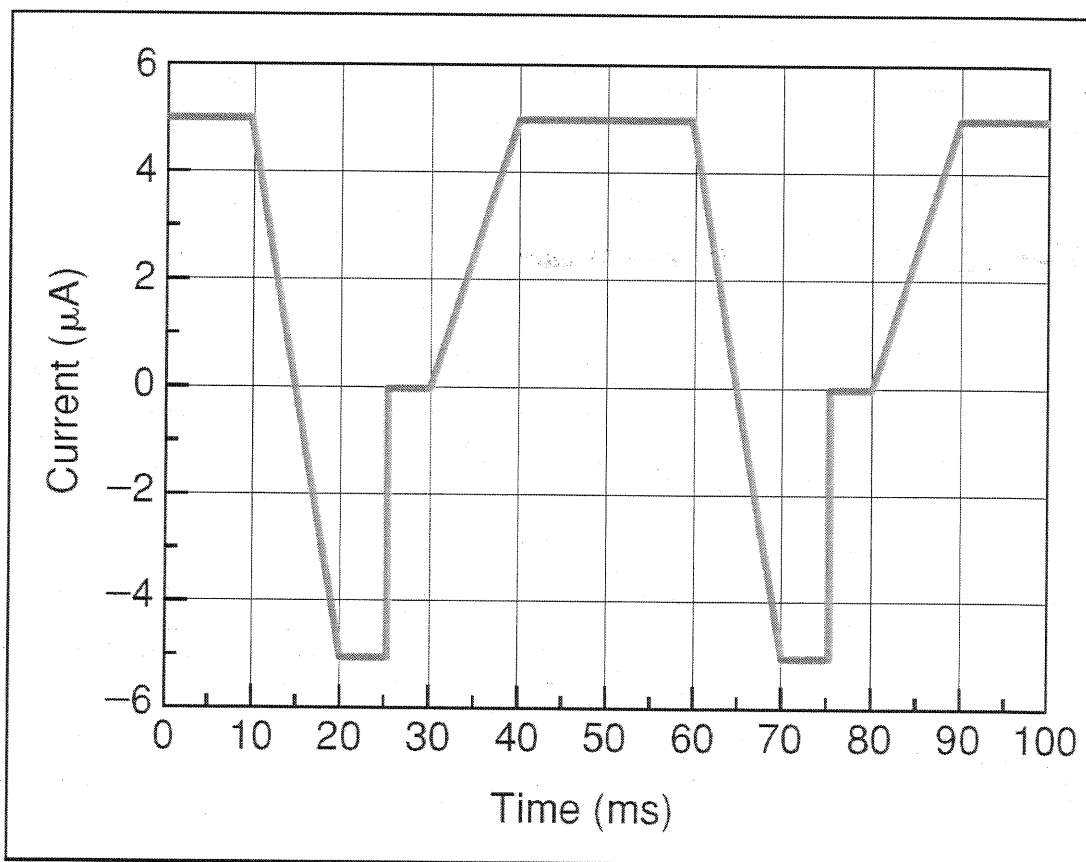


Figure P15.23

SOLUTION:

Final time = 50 ms Stop Ceiling = 50 μ s # harmonics = 10

Use IPWL to create $i_S(t)$

15.23

FOURIER COMPONENTS OF TRANSIENT RESPONSE I(I_Is)

DC COMPONENT = 1.997500E-06

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	2.000E+01	4.401E-06	1.000E+00	1.049E+02	0.000E+00
2	4.000E+01	1.669E-06	3.792E-01	-3.975E+01	-2.495E+02
3	6.000E+01	8.810E-07	2.002E-01	-1.480E+02	-4.627E+02
4	8.000E+01	4.518E-07	1.027E-01	3.521E+01	-3.844E+02
5	1.000E+02	3.183E-07	7.233E-02	1.791E+02	-3.454E+02
6	1.200E+02	3.001E-07	6.819E-02	2.212E+01	-6.073E+02
7	1.400E+02	2.046E-07	4.648E-02	-1.565E+02	-8.907E+02
8	1.600E+02	1.829E-07	4.157E-02	-2.297E+01	-8.621E+02
9	1.800E+02	1.913E-07	4.347E-02	1.625E+02	-7.816E+02
10	2.000E+02	1.592E-07	3.617E-02	-1.800E+00	-1.051E+03

TOTAL HARMONIC DISTORTION = 4.597326E+01 PERCENT

15.24 The discrete line spectrum for a periodic function $f(t)$ is shown in Fig. P15.24. Determine the expression for $f(t)$.

PSV

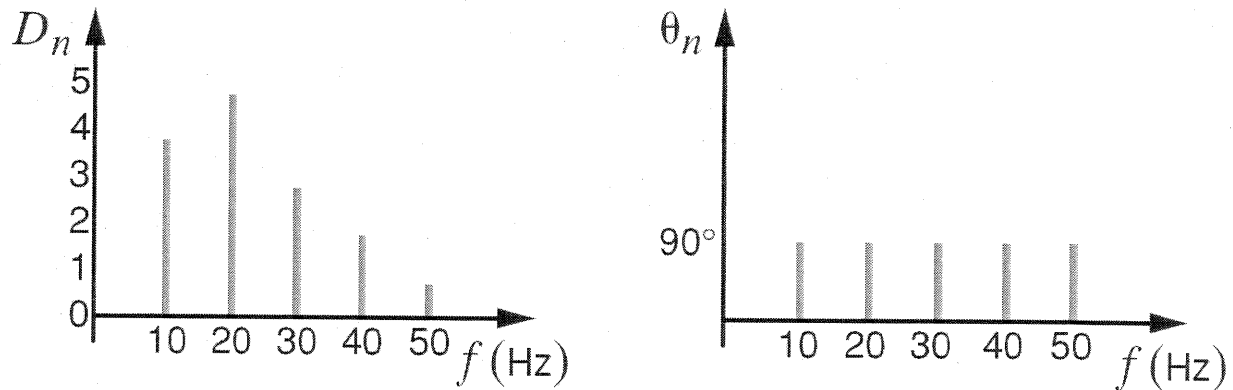


Figure P15.24

SOLUTION: $f_0 = 10 \text{ Hz}$ $\omega_0 = 20\pi \text{ rad/s}$ $D_n = a_n - jb_n$

all $\theta_n = 90^\circ$, so $a_n = 0$ & $b_n = -|D_n|$

$$f(t) = -1 \sin(20\pi t) - 5 \sin(40\pi t) - 3 \sin(60\pi t) - 2 \sin(80\pi t) - \sin(100\pi t) \text{ V}$$

15.25 The amplitude and phase spectra for a periodic function $v(t)$ that has only a small number of terms is shown in Fig. P15.25. Determine the expression for $v(t)$ if $T_0 = 0.1$ s.

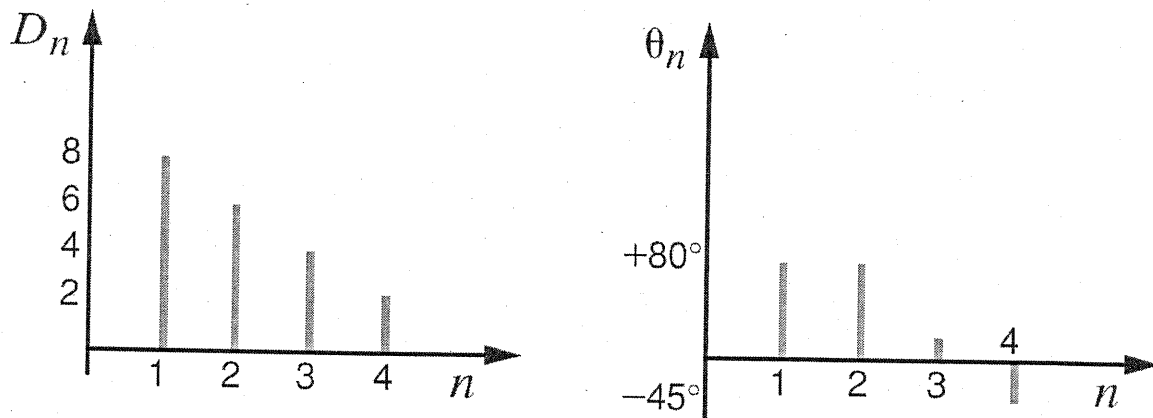


Figure P15.25

SOLUTION: $T_0 = 0.1$ s $f_0 = 10$ Hz $\omega_0 = 20\pi$ rad/s

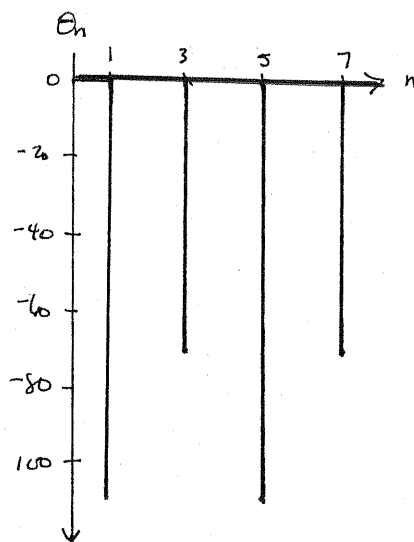
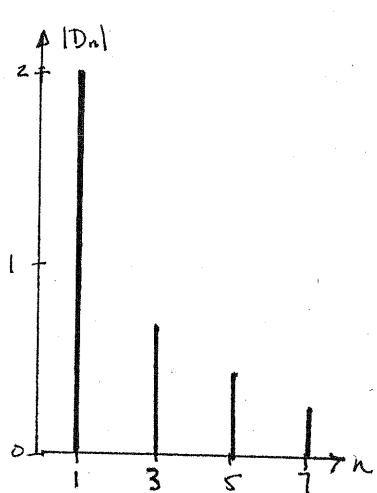
$$v(t) = 8 \cos(20\pi t + 80^\circ) + 6 \cos(40\pi t + 80^\circ) + 4 \cos(60\pi t + 15^\circ) + 2 \cos(80\pi t - 45^\circ)$$

15.26 Plot the first four terms of the amplitude and phase spectra for the signal

$$f(t) = \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{-2}{n\pi} \sin \frac{n\pi}{2} \cos n\omega_0 t + \frac{6}{n\pi} \sin n\omega_0 t$$

SOLUTION: $D_n = a_n - jb_n$

n	a_n	b_n	$ D_n $	$\theta_n (^\circ)$
1	-0.637	1.91	2.01	-108
3	0.212	0.637	0.671	-71.6
5	-0.127	0.382	0.403	-108
7	0.091	0.273	0.288	-71.6



- 15.27** Determine the steady-state response of the current $i_o(t)$ in the circuit shown in Fig. P15.27 if the input voltage is described by the waveform shown in Problem 15.16.

CS

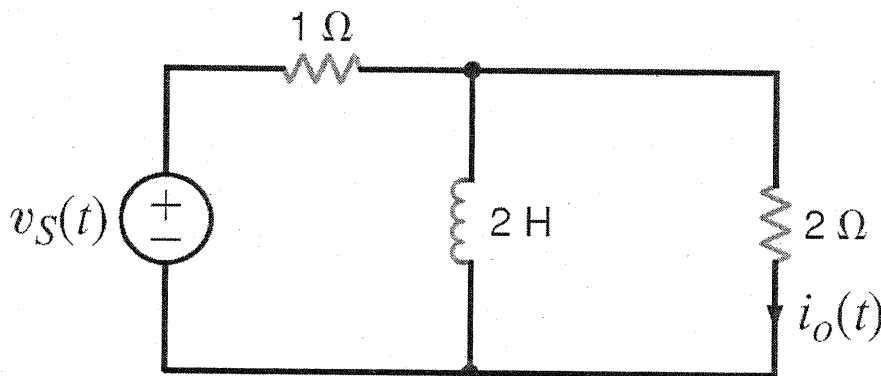
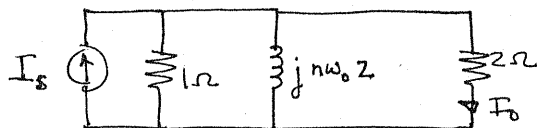


Figure P15.27

SOLUTION:



$$\frac{I_o}{I_s} = \frac{I_o}{V_s} = \frac{1/2}{1/2 + 1 + \frac{1}{j2n}} = \frac{jn}{1 + j3n}$$

$$\text{Let } G(n) = \frac{jn}{1 + j3n} \Rightarrow I_o(n) = G(n) V_s(n)$$

$$i_o(t) = \sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{20}{n\pi} \right) |G(n)| \cos(nt - 90^\circ + \theta_{G(n)}) \text{ A}$$

$$G(n) = \frac{n}{\sqrt{1+9n^2}} \angle 90^\circ - \tan^{-1}(3n)$$

$$i_o(t) = \sum_{n=1}^{\infty} \left[(-1)^{n+1} \left(\frac{20}{n\pi} \right) \frac{n}{\sqrt{1+9n^2}} \cos(nt - \tan^{-1}(3n)) \right] \text{ A}$$

From problem 15.16 $\omega_0 = 1 \text{ rad/s}$

$$V_s = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{20}{n\pi} \cos(nt - 90^\circ) \text{ V}$$

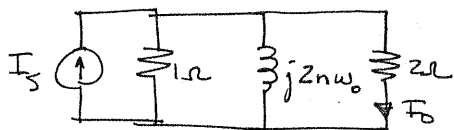
$$I_s = V_s / 1\Omega = V_s$$

15.28 If the input voltage in Problem 15.27 is

$$v_s(t) = 1 - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin 0.2\pi n t \text{ V}$$

find the expression for the steady-state current $i_o(t)$.

SOLUTION:



$$I_s = V_s / 1 = V_s$$

$$\frac{I_o}{I_s} = \frac{E_o}{V_s} = G(n) = |G(n)| \angle \theta(n)$$

$$G(n) = \frac{1/2}{1/2 + 1 + \frac{1}{j2n\omega_0}} = \frac{jn\omega_0}{1 + j3n\omega_0} \quad I_o(n) = G(n) V_s(n)$$

$$v_s(t) = 1 - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \cos(0.2\pi n t - 90^\circ) \quad \omega_0 = 0.2\pi = \pi/5$$

$$G(n) = \frac{jn\pi}{5 + j3n\pi} = |G(n)| \angle \theta(n)$$

for $n=0$, inductor is a short & $i_o \rightarrow 0$.

$$i_o = \sum_{n=1}^{\infty} -\frac{2}{\pi n} |G(n)| \cos\left(\frac{\pi n t}{5} - 90^\circ + \theta(n)\right) \text{ A}$$

15.29 Determine the first three terms of the steady-state voltage $v_o(t)$ in Fig. P15.29 if the input voltage is a periodic signal of the form

$$v(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{n\pi} (\cos n\pi - 1) \sin nt \text{ V} \quad \text{PSV}$$

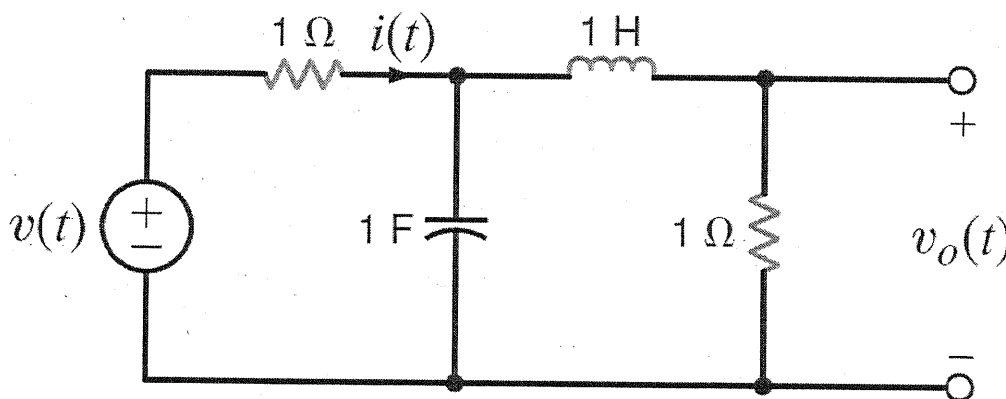
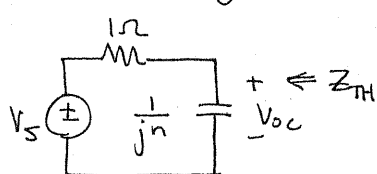


Figure P15.29

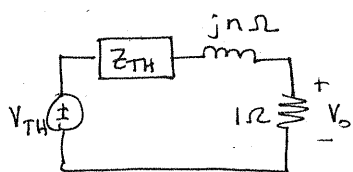
SOLUTION: $\omega_0 = 1 \text{ rad/s}$

Thevenin eq:



$$V_{oc} = \frac{V_s}{1+jn}$$

$$Z_{TH} = \frac{1}{1+jn}$$



$$\frac{V_o}{V_{oc}} = \frac{1}{1+jn + \frac{1}{1+jn}} = \frac{1+jn}{(1+jn)^2 + 1}$$

$$\frac{V_o}{V_s} = \frac{V_o}{V_{oc}} \frac{V_{oc}}{V_s} = \frac{1}{(1+jn)^2 + 1} = \frac{1}{2 - n^2 + j2n} = H(n) = |H(n)| \angle \theta(n)$$

For $n=0$, cap \rightarrow open & inductor \rightarrow short

$$v_o(0) = v(0) \left[\frac{1}{2} \right] = \frac{1}{4} \text{ V}$$

$$\text{for } n=1, \quad v_o(1) = v(1) H(1) = \frac{\cos \pi - 1}{\pi} \frac{1}{1+j2} = 0.285 \angle 116^\circ$$

$$\text{for } n=2, \quad v_o(2) = v(2) H(2) = 0 \quad \text{for } n=3, \quad v_o(3) = 0.023 \angle 41.0^\circ$$

$$v_o(t) = \frac{1}{4} + 0.285 \cos(t + 26.6) + 0.023 \cos(3t - 49^\circ) \text{ V}$$

- 15.30 Determine the steady-state voltage $v_o(t)$ in the network in Fig. P15.30a if the input current is given in Fig. P15.30b.

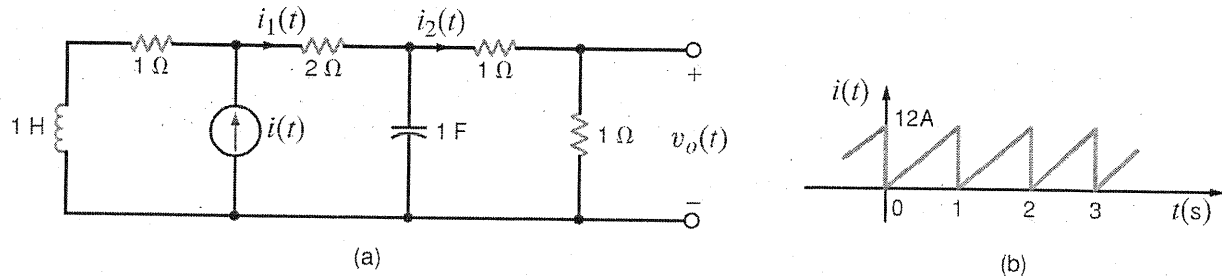
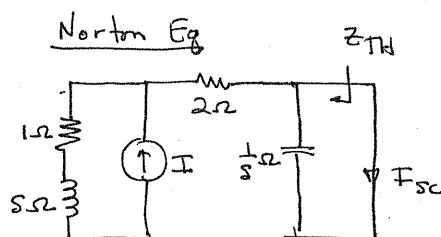


Figure P15.30

SOLUTION: $T_0 = 1\text{ s}$ $\omega_0 = 2\pi\text{ r/s}$

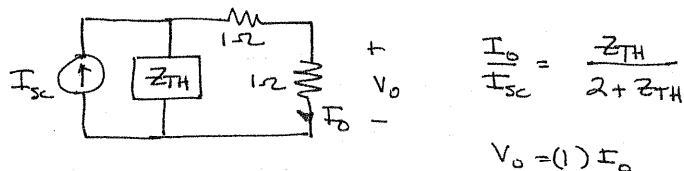
From Table 15.2, $i(t) = 6 + \sum_{n=1}^{\infty} \frac{-12}{n\pi} \sin n2\pi t = 6 + \sum_{n=1}^{\infty} \frac{-12}{n\pi} \cos(n2\pi t - 90^\circ)\text{ A}$



$$I_{sc} = I \left\{ \frac{s+1}{s+3} \right\}$$

$$Z_{TH} = \frac{(1/5)(s+3)}{s+3+1/5}$$

$$Z_{TH} = \frac{s+3}{s^2+3s+1}$$



$$\frac{V_o}{I_{sc}} = \frac{Z_{TH}}{2 + Z_{TH}}$$

$$V_o = (1) I_o$$

$$\frac{V_o}{I_{sc}} = \frac{s+3}{s+3+2(s^2+3s+1)} = \frac{3+s}{(2s+5)(s+1)}$$

$$\frac{V_o}{I} = \frac{s+3}{(2s+5)(s+1)} \cdot \frac{s+1}{s+3} = \frac{1}{2s+5}$$

let $s = j2\pi n$, $\frac{V_o}{I} = \frac{1}{5+j4\pi n} = |G(n)| \angle \theta(n)$

for $n=0$, $i(t) = 6$ & $G(0) = 1/5$

$$V_o(t) = \frac{6}{5} + \sum_{n=1}^{\infty} \frac{-12}{n\pi} |G(n)| \cos(2\pi n t - 90^\circ + \theta(n)) \text{ V} \quad \checkmark$$

15.31 Find the average power absorbed by the network in Fig. P15.31 if

$$v(t) = 12 + 6 \cos(377t - 10^\circ) + 4 \cos(754t - 60^\circ) \text{ V}$$

$$i(t) = 0.2 + 0.4 \cos(377t - 150^\circ)$$

$$-0.2 \cos(754t - 80^\circ) + 0.1 \cos(1131t - 60^\circ) \text{ A}$$

PSV

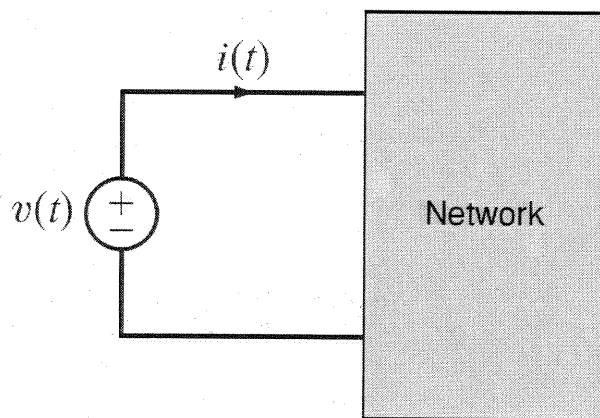


Figure P15.31

SOLUTION:

$$P = V_{DC} I_{DC} + \frac{1}{2} \sum_{n=1}^{\infty} |V_n| |I_n| \cos(\theta_V - \theta_I)$$

$$P = 12(0.2) + \frac{1}{2} \left\{ (6)(0.4) \cos(140^\circ) + 4(0.2) \cos(-160^\circ) \right\}$$

$$P = 1.1 \text{ W}$$

- 15.32** Find the average power absorbed by the network in Fig. P15.32 if $v(t) = 60 + 36 \cos(377t + 45^\circ) + 24 \cos(754t - 60^\circ)$ V. **CS**

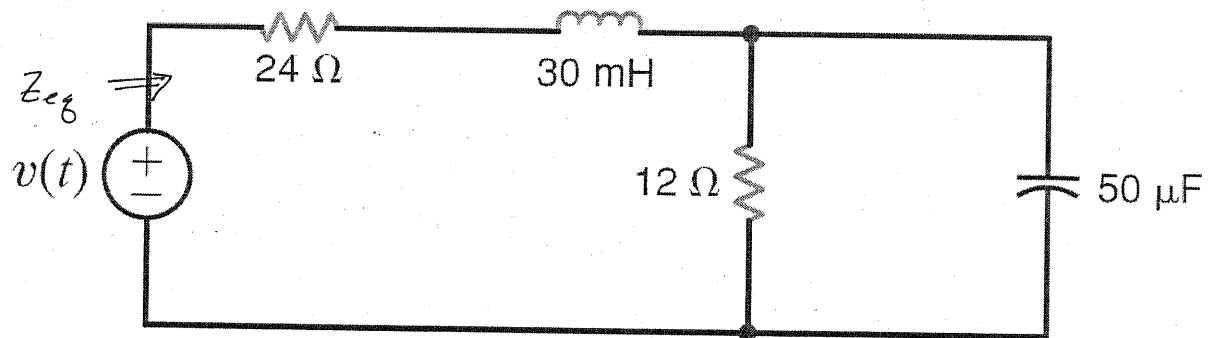


Figure P15.32

SOLUTION: $\omega_0 = 377$ rad/s

$$Z_{eg} = 24 + sL + \frac{12/s}{12 + 1/s} = 24 + sL + \frac{12}{1 + 12s} = \frac{12Lcs^2 + s(288\Omega + L) + 36}{125c + 1}$$

$$\text{Let } s \rightarrow jn\omega_0 = jn377 \quad Z_{eg} = \frac{36 - 2.56n^2 + jn(16.7)}{1 + jn(0.226)}$$

$$I(n) = V(n) / Z_{eg}(n)$$

$$P = \frac{V(0)^2}{Z_{eg}(0)} + \frac{1}{2} \sum_{n=1}^{\infty} \frac{|V(n)|^2}{|Z(n)|} \cos \theta_{Z(n)}$$

$$P = \frac{60^2}{36} + \frac{1}{2} \left\{ \frac{36^2}{36.4} \cos(13.8^\circ) + \frac{24^2}{38.4} \cos(28.1) \right\}$$

$$P = 123.8 \text{ W}$$

15.33 Determine the Fourier transform of the waveform shown in Fig. P15.33.

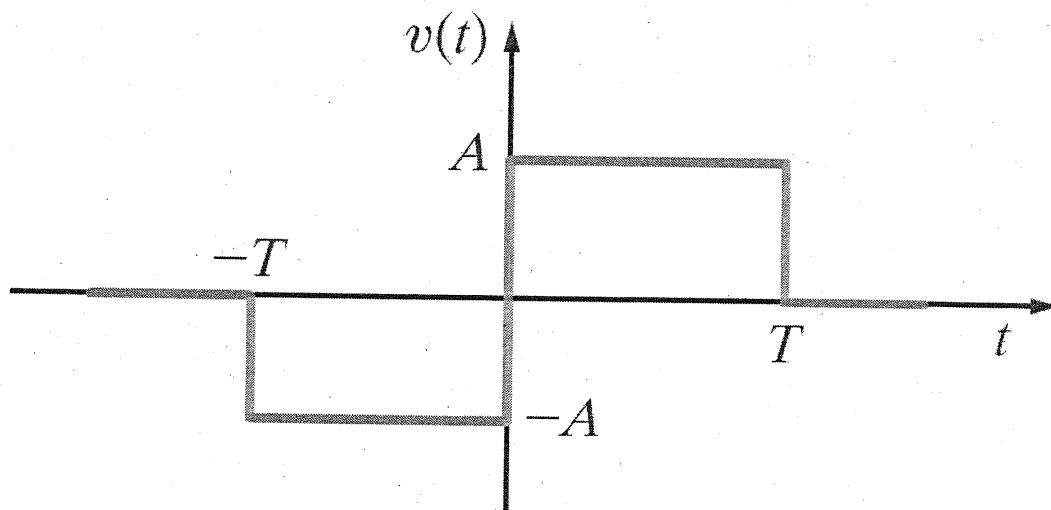


Figure P15.33

SOLUTION:

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt = \int_{-T}^0 -A e^{-j\omega t} dt + \int_0^T A e^{-j\omega t} dt$$

$$F(\omega) = \frac{A}{j\omega} \left[e^{-j\omega t} \Big|_{-T}^0 - e^{-j\omega t} \Big|_0^T \right] = \frac{A}{j\omega} \left[1 - e^{j\omega T} + 1 - e^{-j\omega T} \right]$$

$$F(\omega) = \frac{A}{j\omega} \left[2 - (e^{j\omega T} + e^{-j\omega T}) \right] = \frac{2A}{j\omega} \left[1 - \frac{e^{j\omega T} + e^{-j\omega T}}{2} \right]$$

$$F(\omega) = \frac{2A}{j\omega} \left[1 - \cos(\omega T) \right]$$

15.34 Derive the Fourier transform for the following functions:

(a) $f(t) = e^{-2t} \cos 4t u(t)$

(b) $f(t) = e^{-2t} \sin 4t u(t)$

SOLUTION:

$$a) F(\omega) = \int_0^{\infty} e^{-2t} \cos(4t) e^{-j\omega t} dt = \frac{1}{2} \int_0^{\infty} (e^{-(2-j4+j\omega)t} + e^{-(2+j4+j\omega)t}) dt$$

$$F(\omega) = \frac{1}{2} \left[\frac{e^{-(2-j4+j\omega)t}}{2-j4+j\omega} \Big|_0^{\infty} + \frac{e^{-(2+j4+j\omega)t}}{2+j4+j\omega} \Big|_0^{\infty} \right] = \frac{1}{2} \left[\frac{1}{2-j4+j\omega} + \frac{1}{2+j4+j\omega} \right]$$

$$F(\omega) = \frac{2+j\omega}{(2+j\omega)^2 + 16}$$

$$b) F(\omega) = \int_0^{\infty} e^{-2t} \sin(4t) e^{-j\omega t} dt = \frac{1}{2j} \int_0^{\infty} [e^{-(2-j4+j\omega)t} - e^{-(2+j4+j\omega)t}] dt$$

$$F(\omega) = \frac{1}{2j} \left[\frac{e^{-(2-j4+j\omega)t}}{2-j4+j\omega} \Big|_0^{\infty} - \frac{e^{-(2+j4+j\omega)t}}{2+j4+j\omega} \Big|_0^{\infty} \right] = \frac{1}{2j} \left[\frac{1}{2-j4+j\omega} - \frac{1}{2+j4+j\omega} \right]$$

$$F(\omega) = \frac{1}{j2} \left[\frac{j8}{4+(j\omega)^2 + 4j\omega + 16} \right]$$

$$F(\omega) = \frac{4}{(2+j\omega)^2 + 16}$$

15.35 Show that

$$\mathcal{F}[f_1(t)f_2(t)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbf{F}_1(x)\mathbf{F}_2(\omega - x)dx \quad \text{CS}$$

SOLUTION:

$$\text{Let } G = \frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(x) F_2(\omega - x) dx$$

$$F^{-1}[G] = \frac{1}{(2\pi)^2} \int_{x=-\infty}^{\infty} F_1(x) \int_{\omega=-\infty}^{\infty} F_2(\omega - x) e^{j\omega t} d\omega dx$$

$$\text{Let } u = \omega - x \rightarrow du = d\omega$$

$$F^{-1}[G] = \frac{1}{(2\pi)^2} \int_{x=-\infty}^{\infty} F_1(x) \int_{u=-\infty}^{\infty} F_2(u) e^{j\omega t} e^{jxt} du dx$$

$$F^{-1}[G] = \frac{1}{(2\pi)^2} \int_{x=-\infty}^{\infty} F_1(x) e^{jxt} dx \int_{u=-\infty}^{\infty} F_2(u) e^{j\omega t} du = f_1(t) f_2(t)$$

Thus,

$$F[f_1(t) f_2(t)] = G = \frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(x) F_2(\omega - x) dx \quad \checkmark$$

15.36 Find the Fourier transform of the function

$$f(t) = 12e^{-2|t|} \cos 4t.$$

SOLUTION: Let $g(t) = 12e^{-2|t|}$

From Table 15.3, $G(\omega) = \frac{48}{4+\omega^2}$

$$\cos 4t = \frac{e^{j4t} + e^{-j4t}}{2} \Rightarrow F[g(t) \cos 4t] = F\left[\frac{48}{4+\omega^2} \left(\frac{e^{j4t} + e^{-j4t}}{2}\right)\right]$$

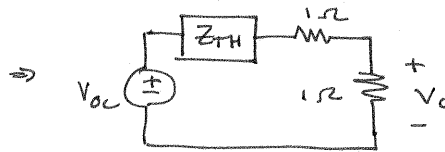
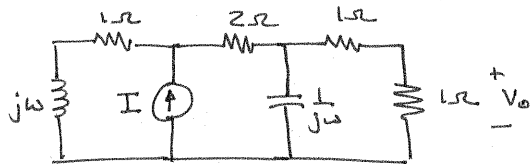
From Table 15.4, $F(\omega) = \frac{1}{2} \left[\frac{48}{4+(\omega-4)^2} + \frac{48}{4+(\omega+4)^2} \right]$

$$F(\omega) = \frac{24}{4+(\omega-4)^2} + \frac{24}{4+(\omega+4)^2}$$

- 15.37** Use the transform technique to find $v_o(t)$ in the network in Fig. P15.30a if (a) $i(t) = 4(e^{-t} - e^{-2t})u(t)$ A and (b) $i(t) = 12 \cos 4t$ A. **PSV**

SOLUTION:

2)



$$I(\omega) = 4 \left[\frac{1}{1+j\omega} - \frac{1}{2+j\omega} \right]$$

$$I(\omega) = \frac{4}{(1+j\omega)(2+j\omega)}$$

$$V_{oc} = I \left[\frac{1+j\omega}{3+j\omega+1/j\omega} \right] \cdot 1/j\omega$$

$$V_{oc} = I \left[\frac{1+j\omega}{1-\omega^2+j3\omega} \right]$$

$$Z_{TH} = \frac{1}{j\omega} \parallel (3+j\omega) = \frac{3+j\omega}{1-\omega^2+j3\omega}$$

$$V_o = V_{oc} \left[\frac{1}{2+Z_{TH}} \right] = \frac{I(1+j\omega)}{5-2\omega^2+j7\omega} = \frac{I}{5+j2\omega}$$

$$H(\omega) = \frac{V_o(\omega)}{I(\omega)} = \frac{1/2}{2.5+j\omega}$$

$$V_o(\omega) = \frac{2}{(1+j\omega)(2+j\omega)(2.5+j\omega)}$$

$$V_o(\omega) = \frac{4/3}{1+j\omega} - \frac{4}{2+j\omega} + \frac{8/3}{2.5+j\omega}$$

$$v_o(t) = \frac{4}{3} e^{-t} - 4 e^{-2t} + \frac{8}{3} e^{-2.5t} u(t)$$

b) $I(\omega) = 12\pi \left[\delta(\omega-4) + \delta(\omega+4) \right]$ $H(\omega) = \frac{1}{5+j2\omega}$

$$V_o(\omega) = 6(2\pi) \left[\frac{\delta(\omega-4)}{5+j8} + \frac{\delta(\omega+4)}{5-j8} \right] = \frac{6(2\pi)}{K} \left[e^{-j\theta} \delta(\omega-4) + e^{j\theta} \delta(\omega+4) \right]$$

$$K = 9.43 \quad \theta = 58^\circ \quad v_o(t) = \frac{6}{K} \left[e^{j(4t-\theta)} + e^{-j(4t-\theta)} \right]$$

$$v_o(t) = \frac{12}{K} \cos(4t-\theta) \text{ V}$$

$$v_o(t) = 1.27 \cos(4t - 58^\circ) \text{ V}$$

15.38 The input signal to a network is $v_i(t) = e^{-3t}u(t)$ V.

The transfer function of the network is

$\mathbf{H}(j\omega) = 1/(j\omega + 4)$. Find the output of the network $v_o(t)$ if the initial conditions are zero. **CS**

SOLUTION:

$$V_i(\omega) = \frac{1}{3+j\omega}$$

$$V_o(\omega) = V_i(\omega) H(\omega) = \frac{1}{(3+j\omega)(4+j\omega)} = \frac{1}{3+j\omega} - \frac{1}{4+j\omega}$$

$$v_o(t) = (e^{-3t} - e^{-4t})u(t) \text{ V}$$

15.39 Determine $v_o(t)$ in the circuit shown in Fig. P15.39 using the Fourier transform if the input signal is $i_s(t) = (e^{-2t} + \cos t)u(t)$ A.

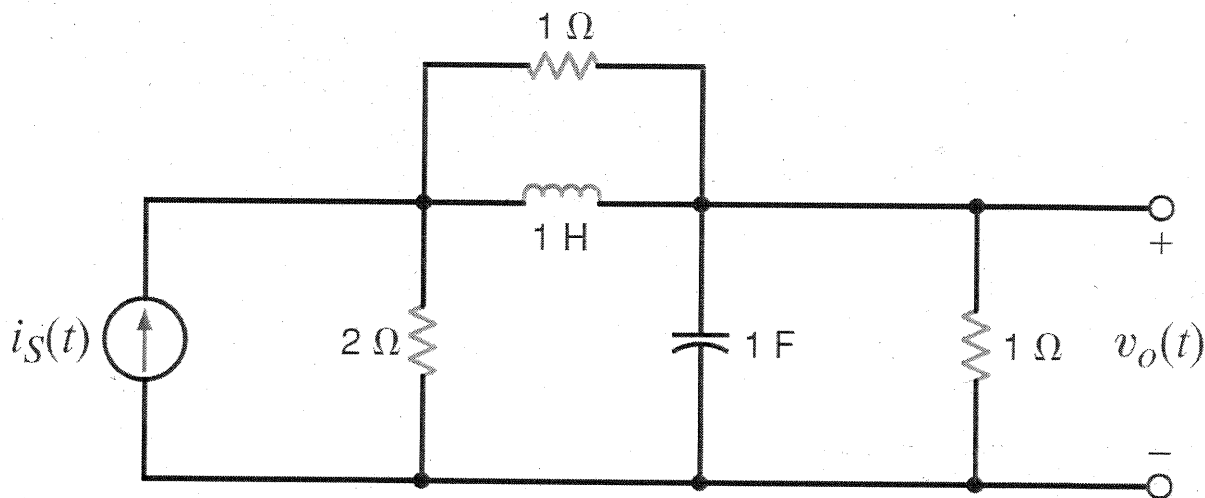


Figure P15.39

SOLUTION: Use source transformation!

$$Z_1 = 1 \parallel j\omega = \frac{j\omega}{1+j\omega}$$

$$Z_2 = 1 \parallel \frac{1}{j\omega} = \frac{1}{1+j\omega}$$

$$\frac{V_o}{2I_s} = \frac{Z_2}{2 + Z_1 + Z_2} = \frac{1}{3(1+j\omega)}$$

$$H(\omega) = \frac{V_o}{I_s} = \frac{2/3}{j\omega + 1}$$

$$I_s = \frac{1}{2+j\omega} + \pi \delta(\omega-1) + \pi \delta(\omega+1)$$

$$V_o(\omega) = \frac{2/3}{(2+j\omega)(1+j\omega)} + \frac{\pi(2/3)\delta(\omega-1)}{1+j1} + \frac{\pi(2/3)\delta(\omega+1)}{1-j1}$$

$$V_o(\omega) = \frac{-2/3}{2+j\omega} + \frac{2/3}{1+j\omega} + \frac{2}{3}\pi \left\{ \frac{\delta(\omega-1)}{1+j1} + \frac{\delta(\omega+1)}{1-j1} \right\}$$

$$v_o(t) = \frac{2}{3} \left[e^{-t} - e^{-2t} + \frac{1}{\sqrt{2}} \cos(t - 45^\circ) \right] u(t) \text{ V}$$

- 15.40** The input signal for the network in Fig. P15.40 is $v_i(t) = 10e^{-5t}u(t)$ V. Determine the total 1- Ω energy content of the output $v_o(t)$.

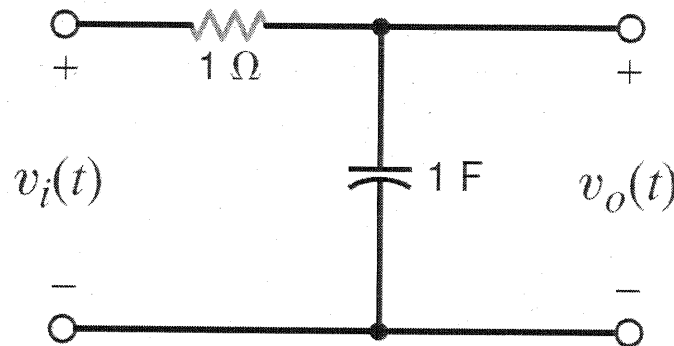


Figure P15.40

SOLUTION:

$$V_i(\omega) = \frac{10}{5 + j\omega} \quad H(\omega) = \frac{1/j\omega}{1 + 1/j\omega} = \frac{1}{1 + j\omega} \quad V_o = \frac{10}{(1 + j\omega)(5 + j\omega)}$$

$$|V_o(\omega)|^2 = \frac{100}{(1 + \omega^2)(25 + \omega^2)} = \frac{100}{24} \left[\frac{1}{1 + \omega^2} - \frac{1}{25 + \omega^2} \right] = \frac{25}{6} \left[\frac{1}{1 + \omega^2} - \frac{1}{25 + \omega^2} \right]$$

$$W = \frac{1}{2\pi} \int_{-\infty}^{\infty} |V_o(\omega)|^2 d\omega = \frac{25}{12\pi} \left\{ \int_{-\infty}^{\infty} \frac{d\omega}{1 + \omega^2} - \int_{-\infty}^{\infty} \frac{d\omega}{25 + \omega^2} \right\}$$

$$W = \frac{25}{12\pi} \left\{ \tan^{-1}(\omega) \Big|_{-\infty}^{\infty} - \frac{1}{5} \tan^{-1}\left(\frac{\omega}{5}\right) \Big|_{-\infty}^{\infty} \right\} = \frac{25}{12\pi} \left[\pi - \frac{\pi}{5} \right] = \frac{25}{12\pi} \left(\frac{4\pi}{5} \right)$$

$$W = \frac{5}{3} \text{ J}$$

15.41 Compute the 1- Ω energy content of the signal $v_o(t)$ in Fig. P15.40 in the frequency range from $\omega = 2$ to $\omega = 4$ rad/s. **CS**

SOLUTION:

From problem 15.40,

$$|V_o|^2 = \frac{25}{6} \left[\frac{1}{1+\omega^2} - \frac{1}{25+\omega^2} \right]$$

$$W = \frac{25}{6\pi} \left[\int_2^4 \frac{d\omega}{1+\omega^2} - \int_2^4 \frac{d\omega}{25+\omega^2} \right] = \frac{25}{6\pi} \left\{ \tan^{-1}(\omega) \Big|_2^4 - \frac{\tan^{-1}(\omega/5)}{5} \Big|_2^4 \right\}$$

$$W = \frac{25}{6\pi} \left\{ 1.326 - 1.107 - \left(\frac{0.675 - 0.381}{5} \right) \right\}$$

$$W = 0.212 \text{ J}$$

15.42 Determine the $1\text{-}\Omega$ energy content of the signal $v_o(t)$ in Fig. P15.40 in the frequency range from 0 to 1 rad/s.

SOLUTION:

From problem 15.40

$$|V_o|^2 = \frac{25}{6} \left[\frac{1}{1+\omega^2} - \frac{1}{25+\omega^2} \right]$$

$$W = \frac{2}{2\pi} \int_0^1 \frac{25}{6} \left(\frac{d\omega}{1+\omega^2} \right) - \frac{2}{2\pi} \int_0^1 \frac{25}{6} \left(\frac{d\omega}{25+\omega^2} \right)$$

$$W = \frac{25}{6\pi} \left[\tan^{-1}(\omega) \Big|_0^1 - \frac{\tan^{-1}(\omega/5)}{5} \Big|_0^1 \right] = \frac{25}{6\pi} [0.785 - 0.039]$$

$$W = 0.990 \text{ J}$$

15.43 Compare the $1\text{-}\Omega$ energy at both the input and output of the network in Fig. P15.43 for the given input forcing function $i_i(t) = 2e^{-4t}u(t)$ A.

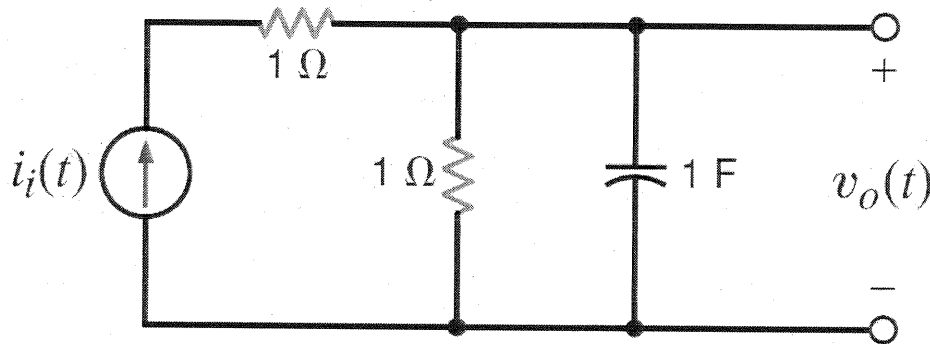
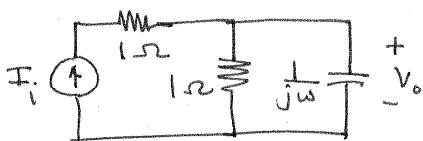


Figure P15.43

SOLUTION:



$$H(\omega) = \frac{V_o(\omega)}{I_i(\omega)} = 1 \parallel \left(\frac{1}{j\omega}\right) = \frac{1}{1+j\omega}$$

$$I_i = \frac{2}{4+j\omega} \quad V_o = \frac{2}{(1+j\omega)(4+j\omega)}$$

$$V_o = \frac{2/3}{1+j\omega} - \frac{2/3}{4+j\omega}$$

$$|I_i|^2 = \frac{4}{16+\omega^2}$$

$$W_{in} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{4}{16+\omega^2} d\omega = \frac{1}{2\pi} \left[\tan^{-1}\left(\frac{\omega}{4}\right) \right] \Big|_{-\infty}^{\infty} = \frac{1}{2} \text{ J}$$

$$|V_o|^2 = \frac{4}{(1+\omega^2)(16+\omega^2)} = \frac{4/15}{1+\omega^2} - \frac{4/15}{16+\omega^2}$$

$$W_{out} = \frac{1}{2\pi} \left(\frac{4}{15}\right) \int_{-\infty}^{\infty} \left(\frac{1}{1+\omega^2} - \frac{1}{16+\omega^2}\right) d\omega = \frac{2}{15\pi} \left[\tan^{-1}(\omega) - \frac{\tan^{-1}(\omega/4)}{4} \right] \Big|_{-\infty}^{\infty}$$

$$W_{out} = \frac{2}{15\pi} \left[\pi - \pi/4 \right]$$

$$W_{out} = 0.1 \text{ J}$$

$$W_{in} = 0.5 \text{ J}$$

- 15.44** The waveform shown in Fig. P15.44 demonstrates what is called the duty cycle; that is, D illustrates the fraction of the total period that is occupied by the pulse. Determine the average value of this waveform.

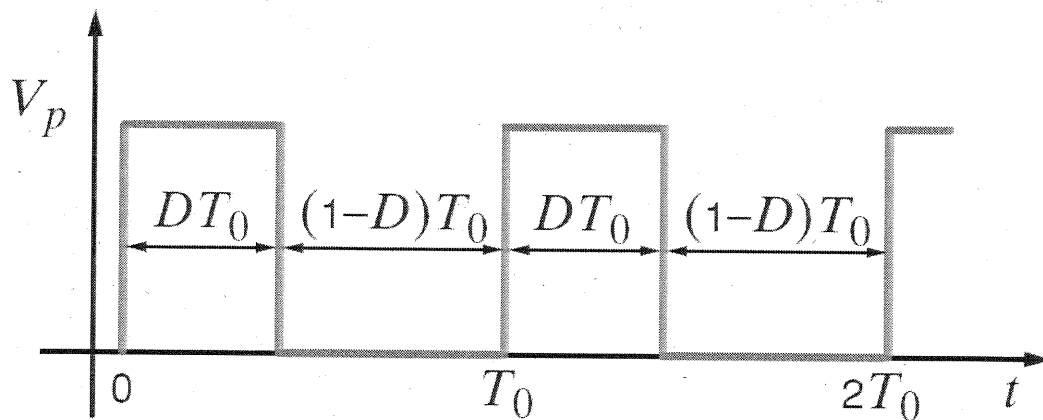


Figure P15.44

SOLUTION:

$$\text{average value} = a_0 = \frac{1}{T_0} \int_0^{T_0} v(t) dt = \frac{1}{T_0} \int_0^{DT_0} V_P dt$$

$$a_0 = \frac{V_P}{T_0} t \Big|_0^{DT_0}$$

$$\boxed{a_0 = V_P D}$$

15FE-1 Given the waveform in Fig. 15PFE-1, determine which of the trigonometric Fourier coefficients have zero value, which have nonzero value, and why.

CS

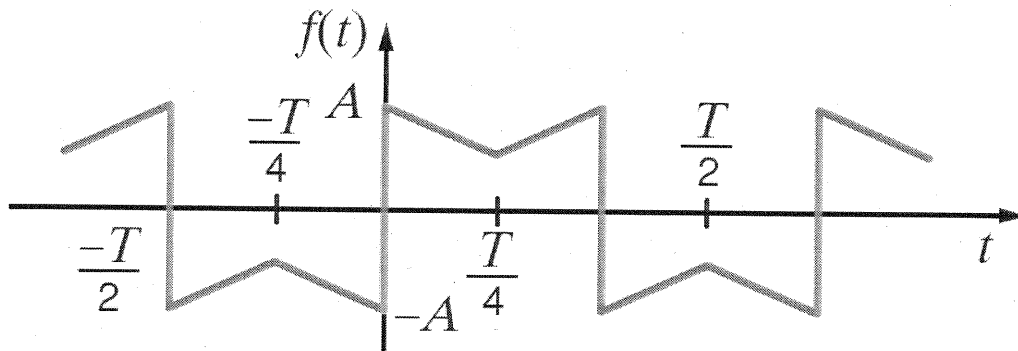


Figure 15PFE-1

SOLUTION:

Average value = 0 $\Rightarrow a_0 = 0$

$a_n = 0$ for all n since waveform has odd symmetry

$b_n = 0$ for n even since waveform has halfwave symmetry

b_n is finite & non zero for n odd.

15FE-2 Given the waveform in Fig. 15PFE-2, describe the type of symmetry and its impact on the trigonometric coefficients in the Fourier series—that is, a_0 , a_n , and b_n .

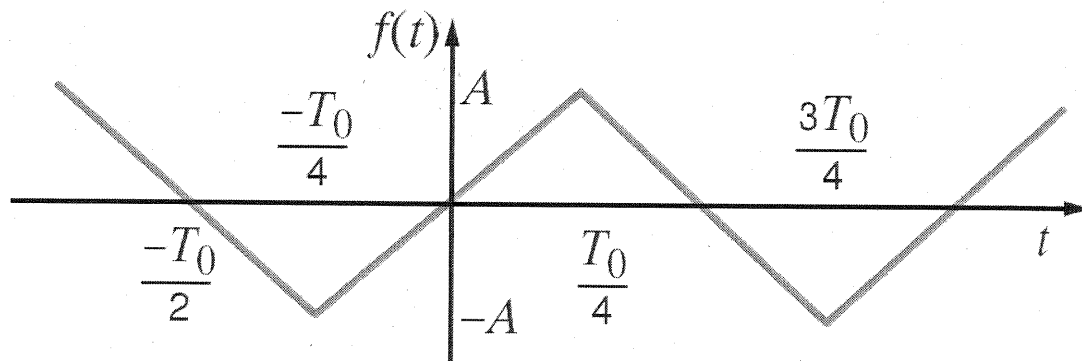


Figure 15PFE-2

SOLUTION:

Average value = 0 $\Rightarrow a_0 = 0$

Odd symmetry \Rightarrow all $a_n = 0$

Half wave symmetry $\Rightarrow b_n = 0$ for n even

b_n is non zero only for n -odd